Overview

- *The Discrete Fourier Transform*
  - *Concept*
  - *Review on the Fourier representation of signals*
    - *case 1: aperiodic continuous-time signal*
    - *case 2: periodic continuous-time signal*
    - *case 3: aperiodic discrete-time signal*
    - *case 4: periodic discrete-time signal*
  - *The discrete Fourier series*
  - *The sampling of the Fourier transform*
  - *The discrete Fourier transform (DFT)*
    - *definition*
    - *Properties of the DFT*
      - *linearity*
      - *circular shift*
      - *duality*
      - *symmetry*
      - *circular convolution*

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The Discrete Fourier Transform (DFT)

- Concept
  - the different faces of the Fourier synthesis/analysis…

$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}[k]e^{jk\frac{2\pi}{N}n}$$

The discrete Fourier transform
(N-periodic in n and k)

$$x[n] = \frac{1}{2\pi}\int_{\omega=-\pi}^{\pi}\tilde{X}\left(e^{j\omega}\right)e^{j\omega n}d\omega$$

The discrete-time Fourier transform
(2π-periodic in ω)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}X(\Omega)e^{j\Omega t}d\Omega$$

The continuous-time Fourier transform
(in t and Ω)

$$\tilde{x}(t) = \frac{1}{2}a_0 + \sum_{k=1}^{+\infty}\left[a_k\cos\left(k\frac{2\pi}{T}t\right) + b_k\sin\left(k\frac{2\pi}{T}t\right)\right]$$

$$\tilde{x}(t) = \frac{1}{T}\sum_{k=-\infty}^{+\infty}c_k\, e^{jk\frac{2\pi}{T}t}$$

The Fourier series
(T-periodic in t)

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The Discrete Fourier Transform (DFT)

- Concept

  - It is an alternative to the Fourier transform or to the Z transform to represent <u>finite sequences</u> describing discrete-time signals and linear time-invariant systems,

  - The DFT is a discrete sequence, while the Fourier transform or the Z transform are functions of continuous variables,

  - the DFT corresponds to a sampling of the Fourier transform using equidistant samples in frequency,

  - the DFT is very important in many signal processing applications because efficient algorithms exist (*e.g.*, the FFT, as we shall see) allowing the fast computation of the DFT, which permits the utilization of the DFT, for example, in real-time spectral analysis applications.

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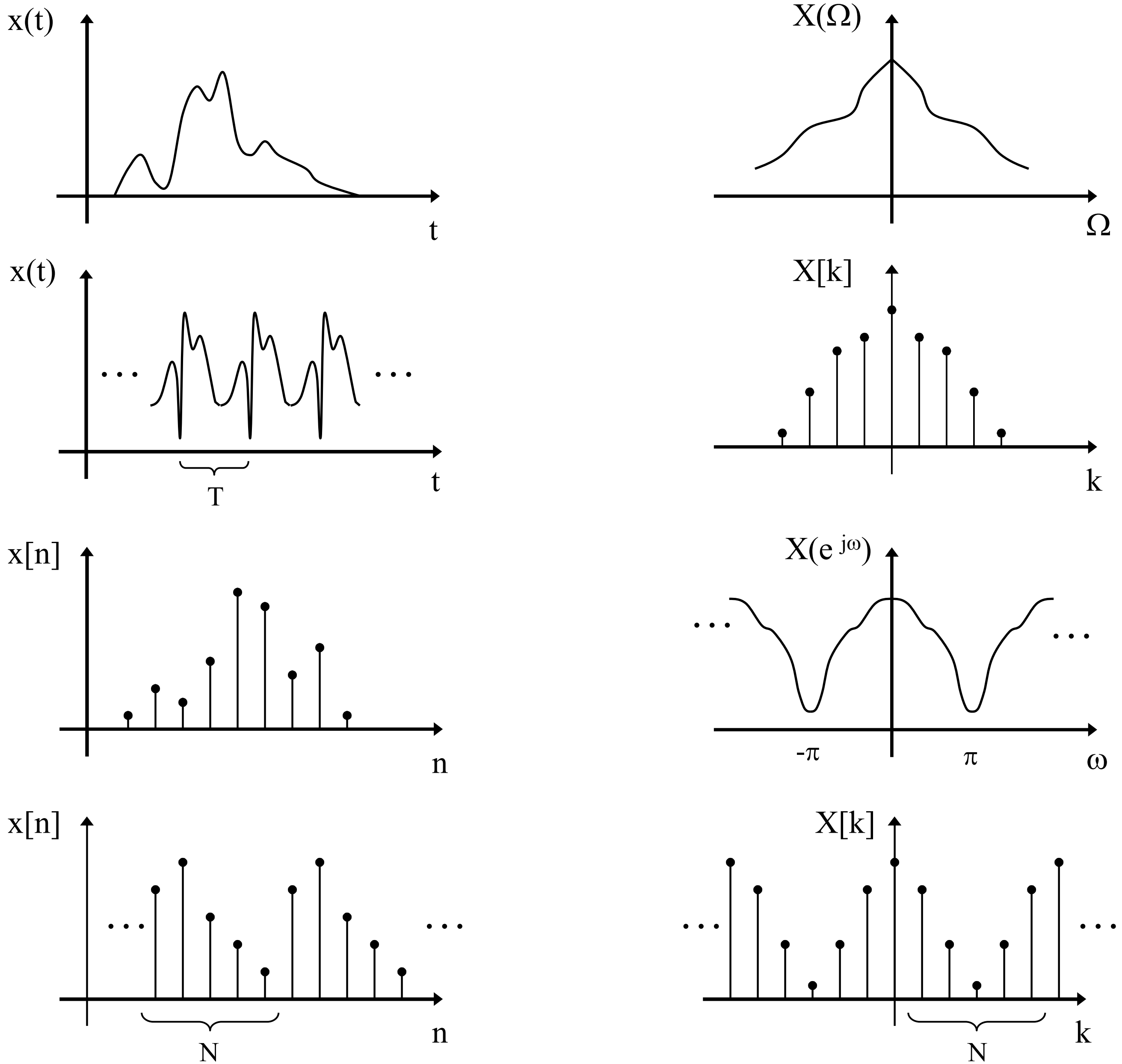
The Fourier representation of signals

→ Case 1

→ Case 2

→ Case 3

→ Case 4



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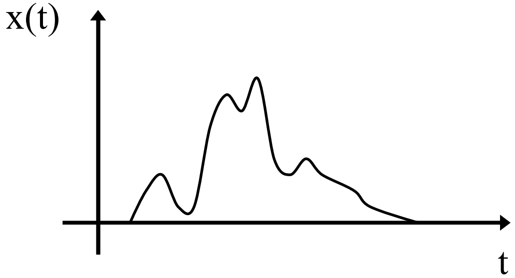
The Fourier representation of signals

- Review on the Fourier representation of signals

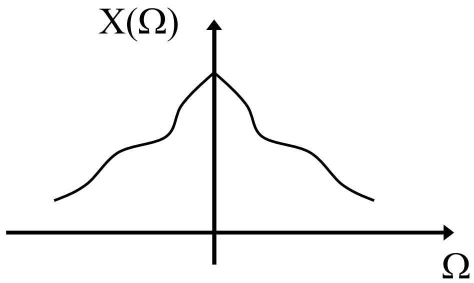
  – we should be familiar already with the Fourier representation of aperiodic continuous-time signals, periodic continuous-time signals, and aperiodic discrete-time signals. The Fourier representation of periodic discrete-time signals is another important case of Fourier representation that consists in the discrete Fourier transform.

$\rightarrow$ Case 1: aperiodic continuous-time signal

- x(t) is aperiodic, X(Ω) is aperiodic.



$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\Omega) e^{j\Omega t} d\Omega$$



$$X(\Omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\Omega t} dt$$

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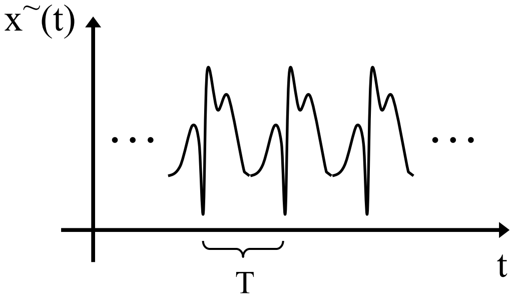
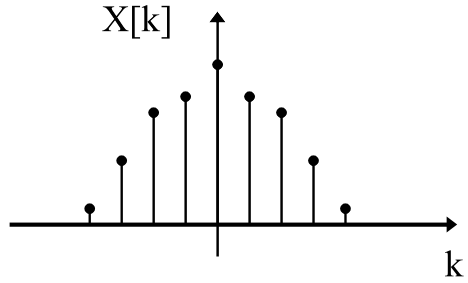
The Fourier representation of signals

→ Case 2: periodic continuous-time signal

- x~(t) is continuous and periodic (with period T),
- its spectrum, X[k], is described by an aperiodic Fourier series, with an infinite number of coefficients that are associated with complex exponentials whose frequencies are multiple integers (*i.e.,* harmonic) of the fundamental frequency $\Omega=2\pi/T$.





$$\widetilde{x}(t) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X[k] e^{jk\frac{2\pi}{T}t}$$

$$X[k] = \int_T \widetilde{x}(t) e^{-jk\frac{2\pi}{T}t} dt$$

$$\widetilde{x}(t) = \widetilde{x}(t+\ell T) \quad , \quad \forall \ell \quad \text{integer}$$

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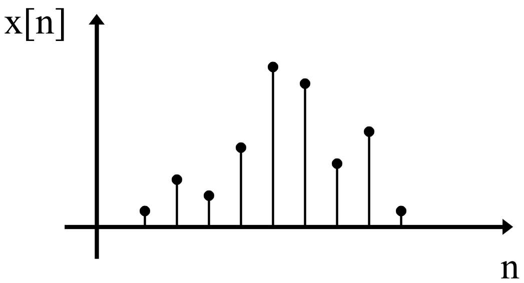
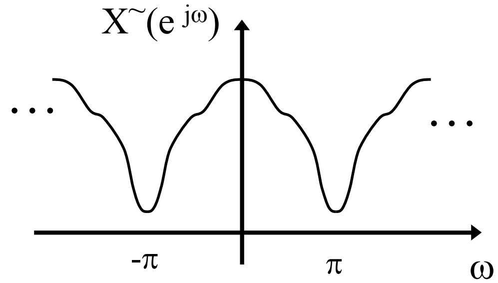
→ Case 3: aperiodic discrete-time signal

- x[n] is aperiodic discrete,
- $\tilde{X}(e^{j\omega})$ is continuous and periodic (with period 2π).





$$x[n] = \frac{1}{2\pi} \int_{2\pi} \tilde{X}\left(e^{j\omega}\right) e^{j\omega n} d\omega$$

$$\tilde{X}\left(e^{j\omega}\right) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$$

$$\tilde{X}\left(e^{j\omega}\right) = \tilde{X}\left(e^{j(\omega+\ell 2\pi)}\right) \quad , \quad \forall \ell \quad \text{integer}$$

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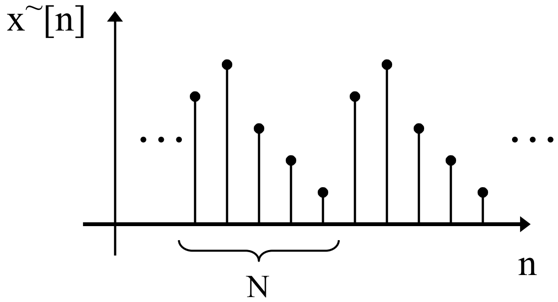
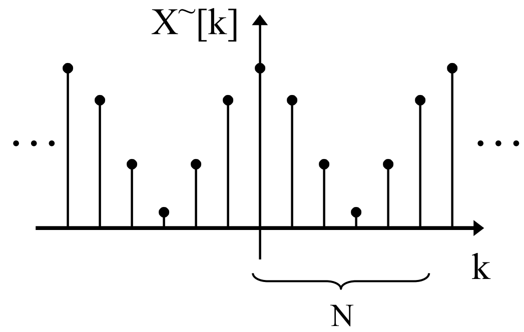
→ Case 4: periodic discrete-time signal

- x~[n] is discrete and periodic (with period N),

- the spectrum of X~[k] is described by an N-periodic Fourier series (N is also the period of the periodic sequence x~[n]) and their coefficients, X~[k], are associated with complex exponentials whose frequencies are harmonic of the fundamental frequency $\omega=2\pi/N$.





$$\boxed{\tilde{x}[n] = \frac{1}{N}\sum_{N} \tilde{X}[k] e^{jk\frac{2\pi}{N}n}}$$

$$\boxed{\tilde{X}[k] = \sum_{N} \tilde{x}[k] e^{-jk\frac{2\pi}{N}n}}$$

$$\boxed{\tilde{x}[n] = \tilde{x}[n + \ell N] \;, \forall\, \ell \in \mathbb{Z}}$$

$$\boxed{\tilde{X}[k] = \tilde{X}[k + \ell N] \quad , \quad \forall \ell \quad \text{integer}}$$

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The Fourier representation of signals

- as a summary…
  - a simple conclusion can be extracted from the four different cases:
    - if the signal is periodic in one domain [time (t or n) or frequency (ω or k)], the signal consists in a set of "lines" in the other domain (frequency or time),
  - the fourth case (periodic Fourier series) is particularly interesting because:
    - it verifies in both domains the two conditions of periodicity and representation using "lines",
    - only N points are necessary in the discrete n domain, or in the discrete frequency domain K, to <u>describe completely a period of the signal</u>.

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The discrete Fourier series

- definition
  - consists in the following Fourier pair that uses N points involving one period of the representation in n, or N points involving one period of the discrete representation in the frequency domain (the tilde symbolizes periodicity):

$$\boxed{\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}[k]W_N^{-kn}} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad \boxed{\tilde{X}[k] = \sum_{n=0}^{N-1}\tilde{x}[n]W_N^{kn}}$$

where: $\boxed{W_N = e^{-j\frac{2\pi}{N}}}$

  - Example: given a periodic signal with period N: $\boxed{\tilde{x}[n] = \sum_{r=-\infty}^{+\infty}\delta[n-rN] = \begin{cases} 1, & n = rN, \quad r \text{ integer} \\ 0, & \text{other} \end{cases}}$

    and given that in a period only one non-zero impulse exists, we have:

$$\boxed{\tilde{X}[k] = \sum_{n=0}^{N-1}\delta[n]W_N^{kn} = W_N^0 = 1} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad \boxed{\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1}W_N^{-kn} = \frac{1}{N}\sum_{k=0}^{N-1}e^{j\frac{2\pi}{N}kn}}$$

but since: $\boxed{\sum_{k=0}^{N-1}e^{j\frac{2\pi}{N}kn} = \begin{cases} N, & \text{n multiple of N} \quad \Leftrightarrow n = \ell N \\ 0, & \textit{other} \end{cases}}$

finally: $\boxed{\tilde{x}[n] = \sum_{\ell=-\infty}^{+\infty}\delta[n-\ell N]}$

NOTE: this result is important to characterize the concept of decimation.

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The sampling of the Fourier transform

- Sampling of the n-discrete Fourier transform
  - there is a *very* important relation between the Fourier series of a periodic discrete signal (in n) with period N, and the Fourier transform of an aperiodic discrete signal whose length is N:
    - sampling the Fourier transform of a discrete-time signal with length N, using N points uniformly distributed (with spacing $2\pi/N$) in the frequency between 0 and 2π, is equivalent to make x[n] periodic with period N.
  - Example:

    Represent the Fourier transform of $x[n]=1,\ 0 \le n \le 4$, and obtain the sequence $\tilde{x}[n]$ that results from sampling $X(e^{j\omega})$ uniformly in frequency using 10 points: $k2\pi/10,\ 0 \le n \le 9$.

A:

$$\boxed{x[n]=\begin{cases}1, & 0\le n\le 4\\ 0, & other\end{cases}} \xleftrightarrow{\ \ F\ \ } \boxed{X\left(e^{j\omega}\right)=e^{-j2\omega}\cdot\frac{\sin(5\omega/2)}{\sin(\omega/2)}}$$

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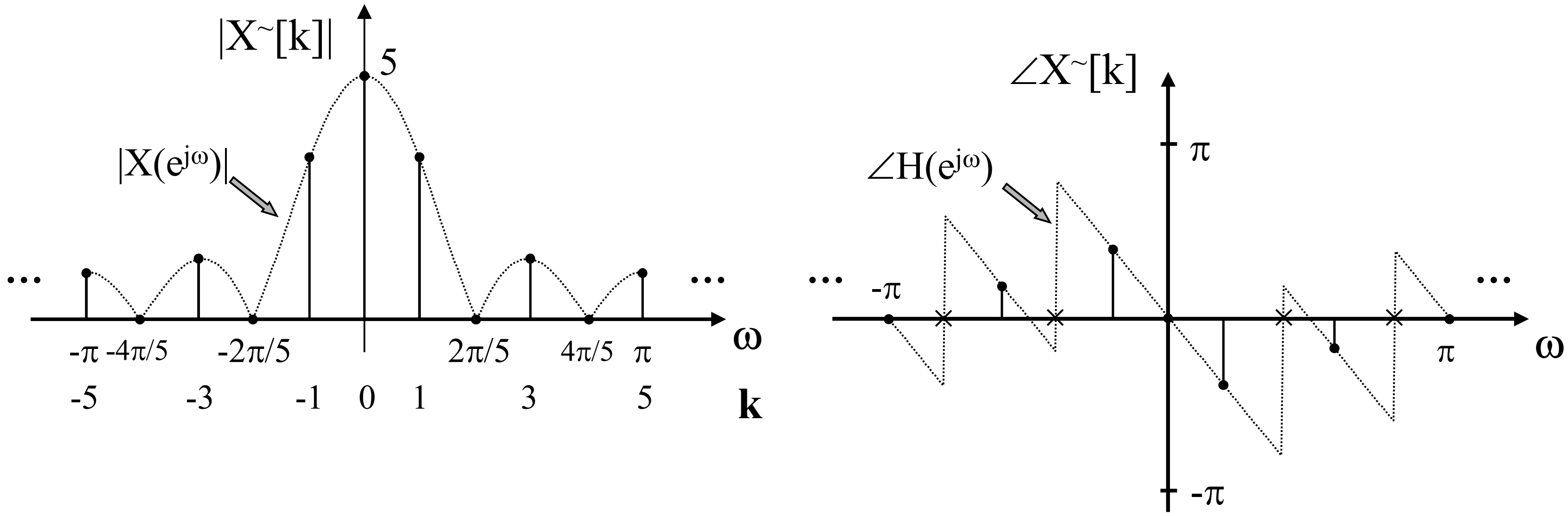
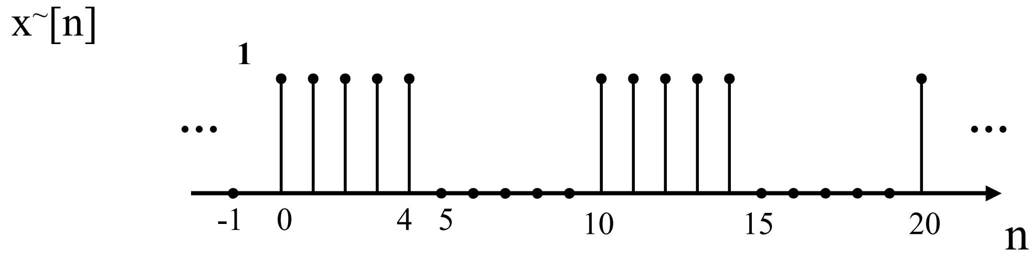
The sampling of the Fourier transform

sampling the Fourier transform we have:



Note: the symbol × in the phase representation means an undefined value since the magnitude is zero.

The sampling of the Fourier transform leads to: $\widetilde{x}[n] = x[n] * \sum_{\ell=-\infty}^{+\infty} \delta[n-10\ell] = \sum_{\ell=-\infty}^{+\infty} x[n-10\ell]$



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The sampling of the Fourier transform

– the result of the previous example may be presented in a more formal way. If x[n] is an aperiodic sequence having Fourier transform $X(e^{j\omega})$, its sampling for $\omega=k2\pi/N$:

$$\widetilde{X}[k] = X\left(e^{j\omega}\right)\Big|_{\omega=k\frac{2\pi}{N}} = X\left(e^{j\frac{2\pi}{N}k}\right)$$

gives rise to a sequence $X^{\sim}[k]$ that is periodic in k, with period N, that may alternatively be obtained using:

$$\widetilde{X}[k] = X(Z)\Big|_{Z=e^{j\frac{2\pi}{N}k}} = X\left(e^{j\frac{2\pi}{N}k}\right)$$

The sequence $X^{\sim}[k]$ may be seen as the Fourier series of a periodic signal $x^{\sim}[n]$ which may be synthesized using a single period of $X^{\sim}[k]$:

$$\widetilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1}\widetilde{X}[k]W_N^{-kn}$$

but since:

$$X\left(e^{j\omega}\right) = \sum_{m=-\infty}^{+\infty} x[m]e^{-j\omega m}$$

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The sampling of the Fourier transform

then: $$\tilde{x}[n]=\frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{m=-\infty}^{+\infty}x[m]e^{-j\frac{2\pi}{N}km}\right]W_N^{-kn}=\sum_{m=-\infty}^{+\infty}x[m]\left[\frac{1}{N}\sum_{k=0}^{N-1}W_N^{-k(n-m)}\right]=\sum_{m=-\infty}^{+\infty}x[m]\tilde{p}[n-m]$$

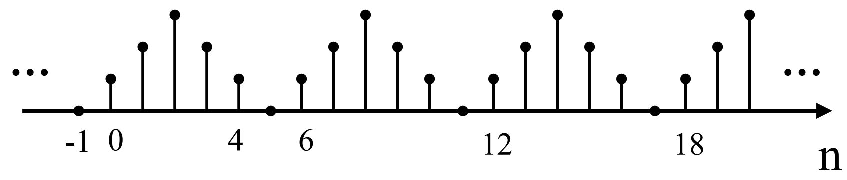
where: $$\tilde{p}[n-m]=\frac{1}{N}\sum_{k=0}^{N-1}W_N^{-k(n-m)}=\frac{1}{N}\sum_{k=0}^{N-1}e^{jk\frac{2\pi}{N}(n-m)}=\begin{cases}1, & n-m=\ell N\\ 0, & other\end{cases}\Leftrightarrow\sum_{\ell=-\infty}^{+\infty}\delta[n-m-\ell N]$$

and finally: $$\tilde{x}[n]=x[n]*\tilde{p}[n]=x[n]*\sum_{\ell=-\infty}^{+\infty}\delta[n-\ell N]=\sum_{\ell=-\infty}^{+\infty}x[n-\ell N]$$

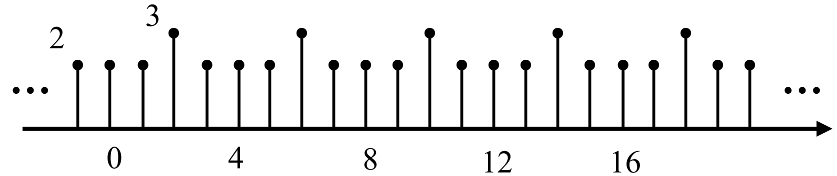
→ we conclude then that <u>sampling the Fourier transform</u> of an aperiodic signal x[n], using N points uniformly distributed in [0, 2π[, <u>gives rise to the superposition</u> of an infinite number of <u>shifted replicas of x[n]</u>. There is however the risk that the superposition in n ("*aliasing* in time") prevents x[n] from being recognized in the periodic sequence, as the following example illustrates:

x[n]

$\sum_{\ell=-\infty}^{+\infty}x[n-6\ell]$



$\sum_{\ell=-\infty}^{+\infty}x[n-4\ell]$



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The sampling of the Fourier transform

- as a summary…
  - the important conclusion that can be extracted from the previous is that in order to recover x[n] from the periodic sequence $\tilde{x}[n]$, it is necessary that the sampling of $X(e^{j\omega})$ be performed using a <u>number of points N that is equal or greater than the length of</u> x[n].
  - if this condition is satisfied, it is possible to recover x[n] from $\tilde{x}[n]$:

$$x[n] = \begin{cases} \tilde{x}[n], & 0 \le n \le N-1 \\ 0, & other \end{cases}$$

  - This discussion is reminiscent of the discussion relative to the uniform sampling of continuous signals:
    - taking a band-limited continuous signal $x_c(t)$, there is no loss of information if instead of being represented for all **t** (continuous), the signal is represented by the samples $x[n]=x_c(nT)$ taken uniformly in time,
  - in similar terms, we may also conclude that:
    - taking a finite length x[n] signal, there is no loss of information if instead of being represented for all ω (continuous), $X(e^{j\omega})$ is represented by **N** uniformly distributed samples in frequency, where N is equal or larger than the length of x[n]. This is the concept underlying the Discrete Fourier Transform (DFT) .

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The Discrete Fourier Transform (DFT)

- Definition
  - consists in the representation of a finite-length discrete sequence, with x[n]≠0, for $0 \leq n \leq N-1$, by N values of x[n] or, equivalently, by N values of its frequency-domain representation X[k], on the basis of the implicit assumption that this discrete frequency representation corresponds, in fact, to the description of a periodic signal, one period of which corresponds to x[n].

→ analysis equation of the DFT: $$X[k]=\sum_{n=0}^{N-1} x[n] W_N^{kn}$$ where: $$W_N = e^{-j\frac{2\pi}{N}}$$

→ synthesis equation of the DFT: $$x[n]=\frac{1}{N}\sum_{k=0}^{N-1} X[k] W_N^{-kn}$$

  - this perspective is of great practical interest (why ?) but we should not forget that in reality and implicitly, we deal with x~[n] and with X~[k], and that we only consider (in order to simplify):

$$x[n]=\begin{cases}\tilde{x}[n], & 0 \leq n \leq N-1 \\ 0, & other \quad n\end{cases}$$

$$X[k]=\begin{cases}\tilde{X}[k], & 0 \leq k \leq N-1 \\ 0, & other \quad k\end{cases}$$

as a summary: periodicity is intrinsic to the definition of the DFT, which naturally constrains its properties.

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The Discrete Fourier Transform (DFT)

- Example: to compute the DFT sequence of length N of the following signal:

$$x[n] = \cos\left(n\ell \frac{2\pi}{N}\right), \quad 0 \le n, \ell \le N-1$$

A: it is easy to conclude that: $x[n] = \frac{1}{2}\left(e^{-j\frac{2\pi}{N}n\ell} + e^{j\frac{2\pi}{N}n\ell}\right) = \frac{1}{2}\left(W_N^{n\ell} + W_N^{-n\ell}\right)$

and as: $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} = \frac{1}{2}\left[\underbrace{\sum_{n=0}^{N-1} W_N^{(k+\ell)n}} + \sum_{n=0}^{N-1} W_N^{(k-\ell)n}\right]$

its value is N for k+ℓ=rN, with r integer, but since 0≤k≤N-1, then there is only one possibility ∴ k=N-ℓ.

it results that: $X[k] = \begin{cases} N/2, & k = \ell \\ N/2, & k = N-\ell \\ 0, & k \in [0, N-1] \setminus \{\ell, N-\ell\} \end{cases}$

NOTE: in this case, there is an alternative way to get to the same result: using the inverse DFT.

QUESTION: how to interpret the case where ℓ=0 ?

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Properties of the DFT

- Linearity

$$\boxed{x_3[n] = ax_1[n] + bx_2[n]} \quad \overset{F}{\longleftrightarrow} \quad \boxed{X_3[k] = aX_1[k] + bX_2[k]}$$

length of $x_1[n] \rightarrow N_1$

length of $x_2[n] \rightarrow N_2$

$\therefore$ length of $x_3[n] \rightarrow N_3 = MAX(N_1, N_2)$

NOTE: the shortest sequence must be extended by appending zeroes (a process that is known as "zero-padding") till it matches the length of the longer sequence, previously to the computation of the DFTs.

- Circular time shift

considering: $\boxed{x[n]} \quad \overset{F}{\longleftrightarrow} \quad \boxed{X[k]}$

we want to know:

$$\boxed{x_1[n]} = ? \quad \overset{F}{\longleftrightarrow} \quad \boxed{e^{-j\frac{2\pi}{N}km} X[k]}$$

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Properties of the DFT

if: $\widetilde{x}[n] = \sum_{\ell=-\infty}^{+\infty} x[n-\ell N] = x([n \text{ modulo } N]) = x([n]_N)$

and as we know that:

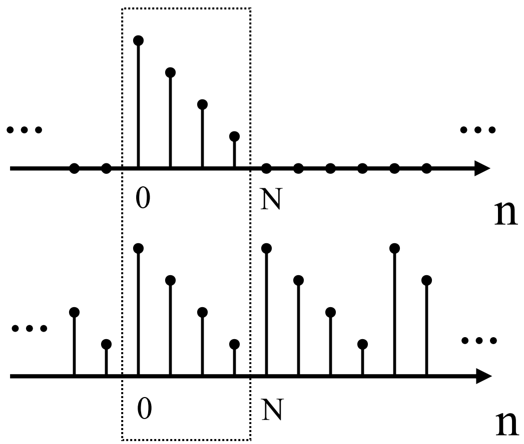
$$\widetilde{x}_1[n] = \widetilde{x}[n-m] \quad \overset{\mathrm{F}}{\longleftrightarrow} \quad \widetilde{X}_1[k] = e^{-j\frac{2\pi}{N}km}\widetilde{X}[k]$$

we conclude that: $x_1[n] = \begin{cases} \widetilde{x}_1[n] = x([n-m]_N), & 0 \le n \le N-1 \\ 0, & other \quad n \end{cases}$

where $x([n\text{-}m]_N)$ represents the circular shift of x[n] as illustrated in the following example where N=4 and m=2:

$x[n]$

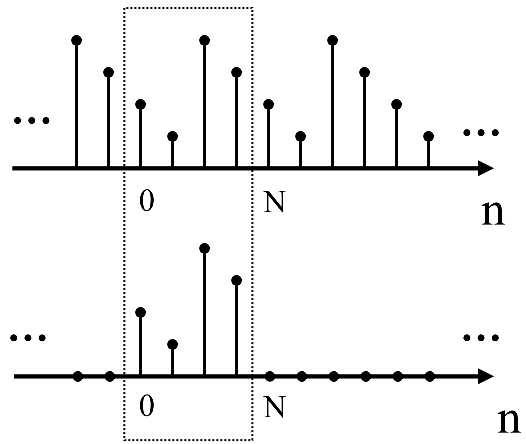
$\widetilde{x}[n]$



$\widetilde{x}[n-2]$

$x_1[n] = x([n-2]_4)$

$0 \le n \le N-1$



NOTE 1: given the nature of the circular shift, then: $x([n-\ell]_N) = x([n+(N-\ell)]_N)$

since: $W_N^{k\ell} = W_N^{-k(N-\ell)}$

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Properties of the DFT

NOTE 2: using a similar procedure, it can also be concluded that :

$$\boxed{e^{j\frac{2\pi}{N}n\ell}x[n]} \xleftrightarrow{\mathrm{F}} \boxed{X([k-\ell]_N)}$$

- Duality

if:

$$\boxed{\tilde{x}[n]} \xleftrightarrow{\mathrm{F}} \boxed{\tilde{X}[k]}$$

it results, considering the duality property of the Fourier series:

$$\boxed{\tilde{X}[n]} \xleftrightarrow{\mathrm{F}} \boxed{N\,\tilde{x}[-k]}$$

and, therefore, if:

$$\boxed{x[n]} \xleftrightarrow{\mathrm{F}} \boxed{X[k]}$$

it results also that:

$$\boxed{X[n]} \xleftrightarrow{\mathrm{F}} \boxed{N\,x([-k]_N), \quad 0 \le k \le N-1}$$

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Properties of the DFT

- Symmetry

defining the following N-length sequences:

$\rightarrow$ periodic conjugate-symmetric component:

$$x_{ep}[n] = \tilde{x}_e[n] = \frac{1}{2}\left(\tilde{x}[n] + \tilde{x}^*[-n]\right) = \frac{1}{2}\left(x[n] + x^*[N-n]\right), \quad 0 \le n \le N-1$$

$\rightarrow$ periodic conjugate-antisymmetric component:

$$x_{op}[n] = \tilde{x}_o[n] = \frac{1}{2}\left(\tilde{x}[n] - \tilde{x}^*[-n]\right) = \frac{1}{2}\left(x[n] - x^*[N-n]\right), \quad 0 \le n \le N-1$$

it results: $\boxed{x[n] = x_{ep}[n] + x_{op}[n]}$

we may also conclude [Oppenheim, section 8.64]:

$$\left.\begin{matrix} \mathfrak{Re}\{x[n]\} \\ j\mathfrak{Im}\{x[n]\} \\ x_{ep}[n] \\ x_{op}[n] \end{matrix}\right\} \xleftrightarrow{\;F\;} \left\{\begin{matrix} X_{ep}[k] \\ X_{op}[k] \\ \mathfrak{Re}\{X[k]\} \\ j\mathfrak{Im}\{X[k]\} \end{matrix}\right.$$

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Properties of the DFT

NOTE : it is also easy to verify that:

$$\left.\begin{array}{r} \boxed{x^*[n]} \\ \boxed{x^*([-n]_N)} \end{array}\right\} \overset{\mathsf{F}}{\longleftrightarrow} \left\{\begin{array}{l} \boxed{X^*([-k]_N)} \\ \boxed{X^*[k]} \end{array}\right.$$

- Circular convolution

If $x_1[n]$ and $x_2[n]$ are two N-length sequences whose DFTs are $X_1[k]$ and $X_2[k]$, respectively, what is $x_3[n]$, the inverse DFT of the product $X_1[k]X_2[k]$?

A: Considering the periodic sequences $\boxed{\tilde{x}_1[n] = x_1([n]_N)}$ and $\boxed{\tilde{x}_2[n] = x_2([n]_N)}$ then:

$$\boxed{\tilde{x}_3[n] = \tilde{x}_1[n] * \tilde{x}_2[n] = \sum_{\ell=0}^{N-1} \tilde{x}_1[\ell]\tilde{x}_2[n-\ell]}$$

which is the periodic convolution.

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Properties of the DFT

Using this result it is also :

$$x_3[n] = \sum_{\ell=0}^{N-1} x_1\left([\ell]_N\right) x_2\left([n-\ell]_N\right), \quad 0 \le n \le N-1$$

which may be expressed as:

$$x_3[n] = \sum_{\ell=0}^{N-1} x_1[\ell] x_2\left([n-\ell]_N\right) = x_1[n] \otimes x_2[n], \quad 0 \le n \le N-1$$

The notation $x_1[n] \otimes x_2[n]$ is representative of the <u>circular convolution</u> because, in its computation, the second sequence is inverted in ℓ and is circularly shifted relative to the length of its period.

NOTE 1: differently from the linear convolution, the result of the circular convolution between two N-length sequences has length N.

NOTE 2: the circular convolution is also commutative and hence:

$$x_1[n] \otimes x_2[n] = x_2[n] \otimes x_1[n] = \sum_{\ell=0}^{N-1} x_2[\ell] x_1\left([n-\ell]_N\right), \quad 0 \le n \le N-1$$

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Properties of the DFT

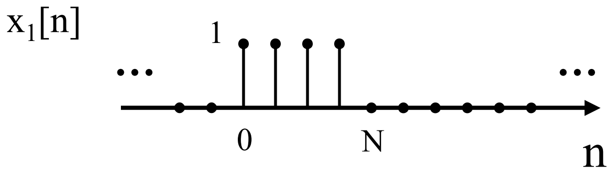
- Example 1:

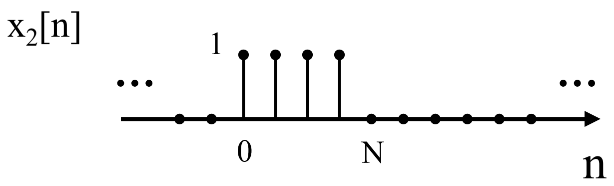
If: $x_1[n] = x_2[n] = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & outros \quad n \end{cases}$ what is the result of $x_1[n] \otimes x_2[n]$?

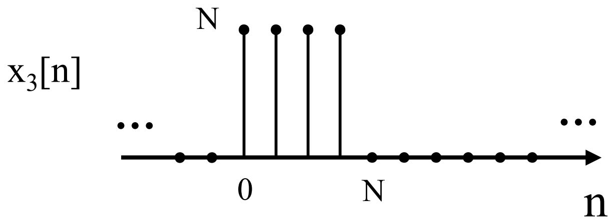
A: as: $X_1[k] = X_2[k] = \sum_{n=0}^{N-1} W_N^{kn} = \begin{cases} N, & k = 0 \\ 0, & 1 \le k \le N-1 \end{cases}$

then: $X_3[k] = X_1[k] X_2[k] = \begin{cases} N^2, & k = 0 \\ 0, & 1 \le k \le N-1 \end{cases} \quad \xleftrightarrow{F} \quad x_3[n] = N, \quad 0 \le n \le N-1$

graphically we have (*e.g.,* N=4):







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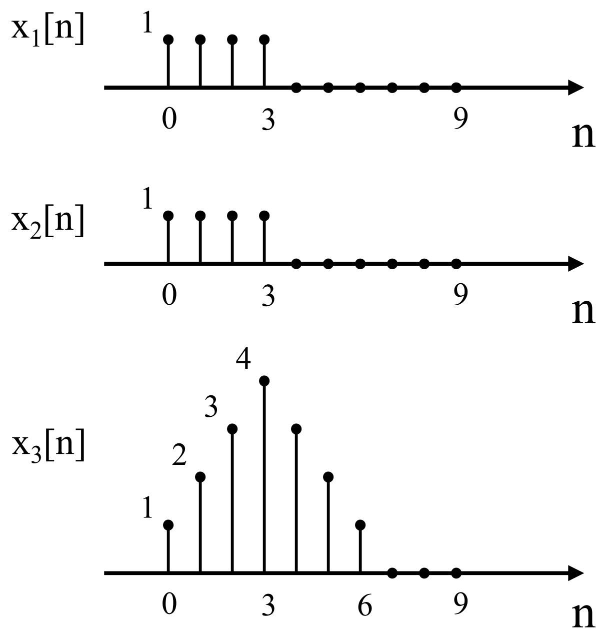
- Example 2:

If: $x_1[n] = x_2[n] = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & L \le n \le N-1 \cup n < 0 \cup n > N \end{cases}$ what is the result of $x_1[n] \otimes x_2[n]$?

A: as: $X_1[k] = X_2[k] = \sum_{n=0}^{L-1} W_N^{kn} = \dfrac{1-W_N^{kL}}{1-W_N^{k}}$

then: $X_3[k] = X_1[k] X_2[k] = \left(\dfrac{1-W_N^{kL}}{1-W_N^{k}} \right)^2$

admitting N=10 and L=4, graphically we have:



Question 1: May we state that in this example the result of the circular convolution is the same as that of the linear convolution ?

Question 2: Keeping L=4, what is the minimum value of N that leads to the same result ?

Question 3: May we state that we may use the circular convolution to compute the linear convolution ? If yes, under which conditions ?

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Properties of the DFT

– It can also be shown that:

$$\boxed{x_1[n]\cdot x_2[n]} \quad \xleftrightarrow{\;F\;} \quad \boxed{\frac{1}{N}X_1[k]\otimes X_2[k]=\frac{1}{N}\sum_{\ell=0}^{N-1}X_1[\ell]X_2\big([k-\ell]_N\big)}$$

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