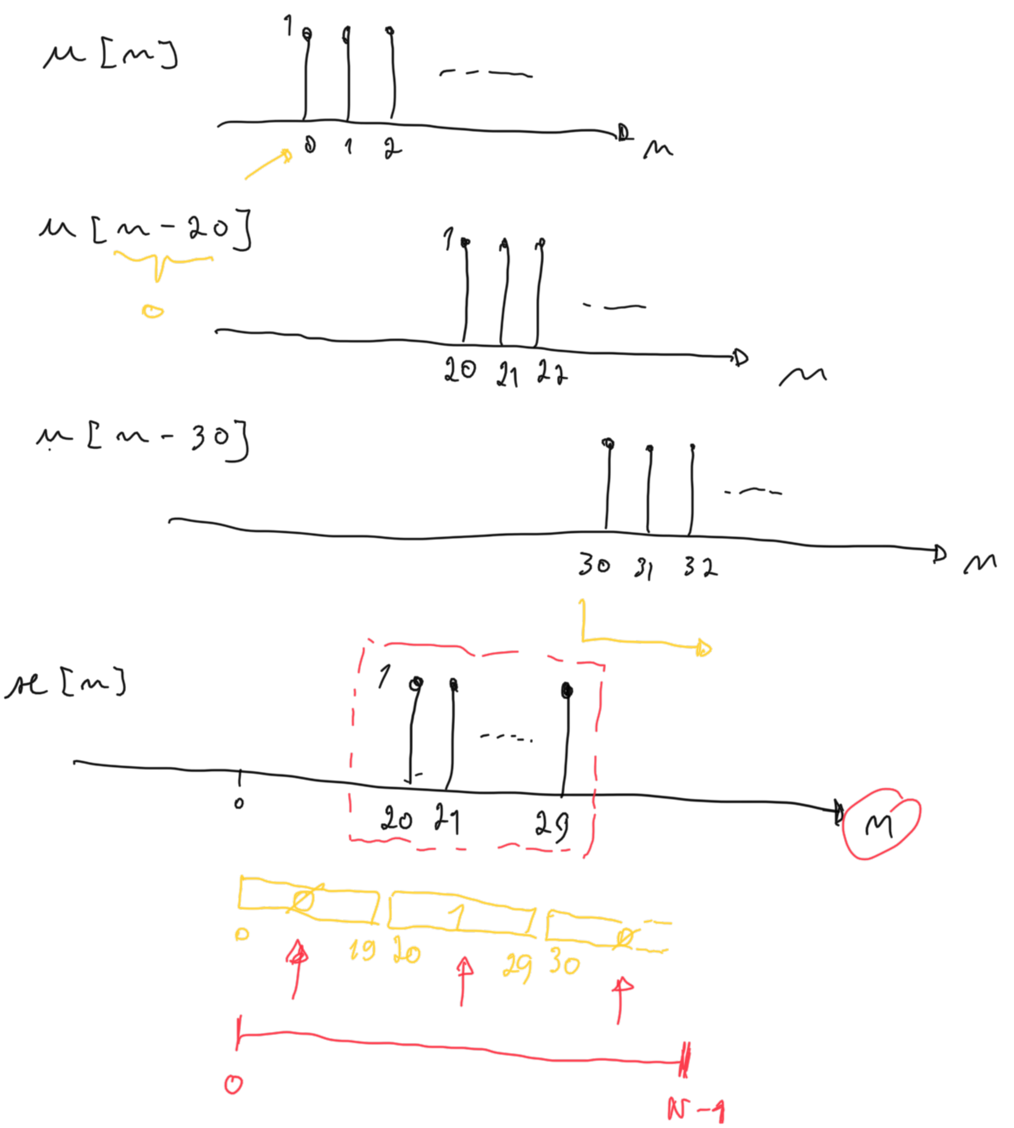
PDSI_TP18FEB2021

$$x[n] = u[n-20] - u[n-30]$$



3

$$h[n] = 2^{-n} u[n]$$

$$x[n] = u[n] - u[n-10]$$

$$y[n] = \boxed{h[n] * x[n]} \triangleq \boxed{\sum_{k=-\infty}^{+\infty} h[k]\, x[n-k]}$$

$$= x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$$h[k] = 2^{-k} u[k]$$

$x[n-k]$

$x[-(k-n)]$

$x[k]$

$x[k-n]$

$x[-k]$

$$x[-\underbrace{(k-n)}_{0}] = x[n-k]$$

$\boxed{n<0}$ $\quad y[n] = 0$

$$\boxed{\sum_{k=a}^{b} r^k = \frac{r^a - r^{b+1}}{1-r}}$$

$n-9 = 0 \Leftrightarrow n = 9$

$\boxed{0 \le n \le 9}$

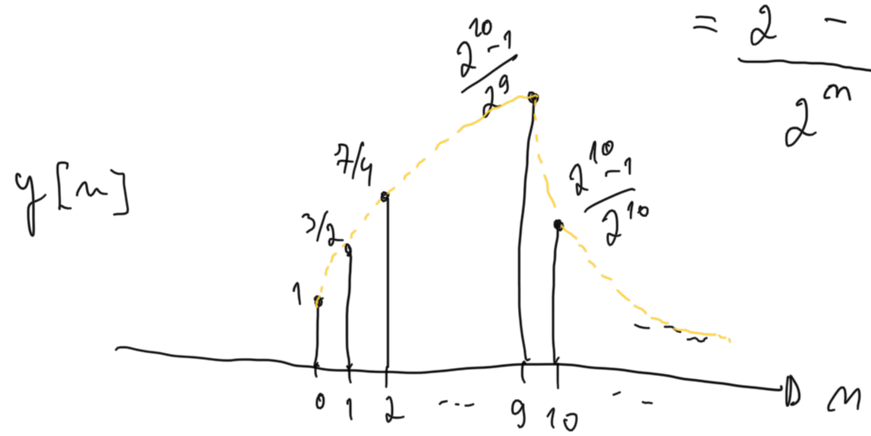
$$y[n] = \sum_{k=0}^{n} 2^{-k} = \frac{1 - 2^{-(n+1)}}{1 - \frac{1}{2}} = 2\left(1 - 2^{-n} 2^{-1}\right)$$

$$= 2 - 2^{-n} = \frac{2^{n+1} - 1}{2^n}$$

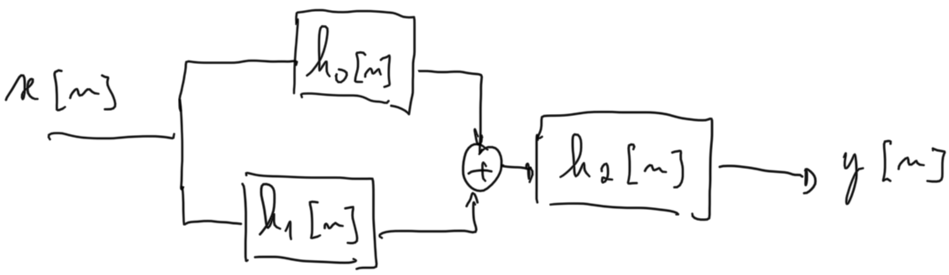
$\boxed{n \geq 9}$

$$y[n] = \sum_{k=n-9}^{n} 2^{-k} = \frac{2^{-(n-9)} - 2^{-(n+1)}}{1 - \frac{1}{2}} = 2\left(2^{+9} 2^{-n} - 2^{-n} 2^{-1}\right)$$

$$= \frac{2^{10} - 1}{2^n}$$



5



$h_1[n] = \beta\,\delta[n-1]$

$h_2[n] = \alpha^n u[n]$

$h_0[n] = \delta[n]$

a) overall $h[n]$

b) overall $H(e^{j\omega})$

c) dif. eq.

a) $h[n] = \left(h_0[n] + h_1[n]\right) * h_2[n]$

$= \left(\delta[n] + \beta\,\delta[n-1]\right) * \alpha^n u[n]$

$= \alpha^n u[n] + \beta\,\alpha^{n-1} u[n-1]$

b) $h_0[n] = \delta[n] \longleftrightarrow H_0(e^{j\omega}) = 1$

$h_1[n] = \beta\,\delta[n-1] \longleftrightarrow H_1(e^{j\omega}) = \beta e^{-j\omega}$

$h_2[n] = \alpha^n u[n] \longleftrightarrow H_2(e^{j\omega}) = \dfrac{1}{1-\alpha e^{-j\omega}}$

$H(e^{j\omega}) = \left(H_0(e^{j\omega}) + H_1(e^{j\omega})\right) \cdot H_2(e^{j\omega})$

$= \dfrac{1+\beta e^{-j\omega}}{1-\alpha e^{-j\omega}}$

$\mathcal{F}$

c) $H(e^{j\omega}) \triangleq \boxed{\dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1+\beta e^{-j\omega}}{1-\alpha e^{-j\omega}}}$

$Y(e^{j\omega})\left(1-\alpha e^{-j\omega}\right) = X(e^{j\omega})\left(1+\beta e^{-j\omega}\right)$

$Y(e^{j\omega}) - \alpha e^{-j\omega} Y(e^{j\omega}) = X(e^{j\omega}) + \beta e^{-j\omega} X(e^{j\omega})$

$\downarrow \mathcal{F}^{-1}$

$y[n] - \alpha\, y[n-1] = x[n] + \beta\, x[n-1]$

$$y[n] = x[n] + \beta x[n-1] + \alpha y[n-1]$$