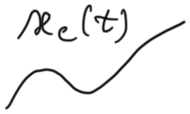
PDSi18FEB2021

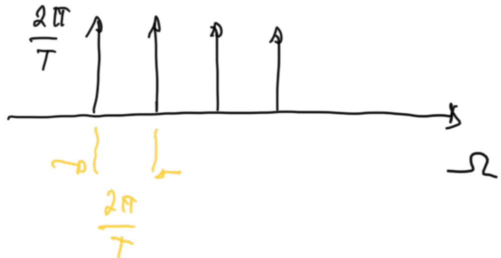
3rd lecture.

$x_c(t)$

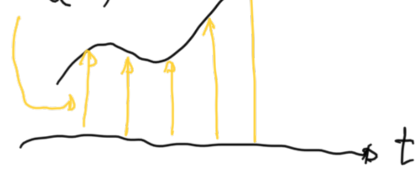


$$\longrightarrow \quad x_a(t) = x_c(t)\, s(t)$$

$$s(t) \xleftrightarrow{\mathcal{F}} S(\Omega)$$



$$\boxed{s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT)} \xleftrightarrow{\mathcal{F}} S(\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\Omega - \frac{2\pi}{T}k\right)$$



$$\boxed{x_a(t) = x_c(t) \cdot s(t)} \xleftrightarrow{\mathcal{F}} X_a(\Omega) = \frac{1}{2\pi} X_c(\Omega) * S(\Omega)$$

$$x_c(t) \xleftrightarrow{\mathcal{F}} X_c(\Omega)$$

$$s(t) \longleftrightarrow S(\Omega)$$

$$X_a(\Omega) = \frac{1}{\cancel{2\pi}} X_c(\Omega) * \frac{\cancel{2\pi}}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\Omega - k\frac{2\pi}{T}\right)$$

$$= \boxed{\frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\Omega - k\frac{2\pi}{T}\right)}$$

$$\boxed{\quad k=-\infty \quad}$$

$$X_a(\Omega) \triangleq \int_{-\infty}^{+\infty} x_a(t)\, e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{+\infty} \underbrace{x_c(t)\cdot \sum_{n=-\infty}^{+\infty} \delta(t-nT)}_{\sum_{n=-\infty}^{+\infty} x_c(nT)\,\delta(t-nT)}\, e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x_c(nT)\, \underbrace{\delta(t-nT)\, e^{-j\Omega t}}_{e^{-j\Omega nT}}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} x_c(nT) \int_{-\infty}^{+\infty} \delta(t-nT)\, e^{-j\Omega nT}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} \underbrace{x_c(nT)}_{x[n]}\, e^{-j\Omega nT} \qquad \Omega T = \omega$$

rad/s

$$\boxed{\omega = \Omega T}$$

rad.

$$= \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\omega n} \triangleq X(e^{j\omega})$$

$\therefore$

$$X(e^{j\omega}) = X_a(\Omega)\big|$$

$$X(e^{j\omega}) = X_a(\Omega)\Big|_{\Omega = \frac{\omega}{T}}$$

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\Omega - k\frac{2\pi}{T}\right)\Bigg|_{\omega = \Omega T}$$

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\frac{\omega - k2\pi}{T}\right)$$