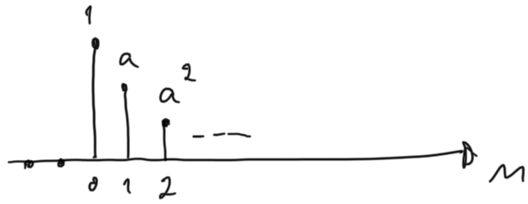
PDSI25FEB2021

Lecture #5

$x[n] = a^n u[n] \,, \quad |a| < 1$



$$X(z) \triangleq \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}$$

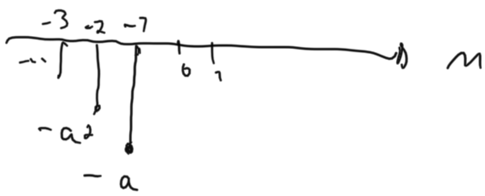
$$= \sum_{n=0}^{+\infty} a^n z^{-n} = \sum_{n=0}^{+\infty} \left(a z^{-1}\right)^n$$

$$\boxed{\sum_{k=a}^{b} r^k = \frac{r^a - r^{b+1}}{1 - r}}$$

$$= \frac{1 - \left(a z^{-1}\right)^{+\infty + 1}}{1 - a z^{-1}} = \frac{1}{1 - a z^{-1}} \,, \quad |z| > |a| = RoC$$

$|a z^{-1}| < 1 \;\therefore\; |z| > |a|$

$x[n] = -a^n u[-n-1]$



$$X(z) \triangleq \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{-1} -a^n z^{-n} = -\sum_{n=-\infty}^{-1} \left(a z^{-1}\right)^n =$$

$n = -m$

$$= -\sum_{m=1}^{+\infty} \left(a z^{-1}\right)^{-m} = -\sum_{m=1}^{+\infty} \left(a^{-1} z\right)^{m}$$

$$= -\frac{a^{-1}z - \left(a^{-1}z\right)^{+\infty+1}}{1 - a^{-1}z} = -\frac{a^{-1}z}{1 - a^{-1}z}$$

$|a^{-1}z| < 1 \quad \therefore \quad |z| < |a|$

$$= -\frac{1}{a z^{-1} - 1} = \frac{1}{1 - a z^{-1}}, \quad |z| < |a| \equiv RoC$$

$$= \frac{z}{z - a}, \quad |z| < |a|$$