FPS 08 Nov 2021

Lecture

$$x[n] = a^n u[n]$$

$$X(z) \triangleq \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$$

$$= \sum_{n=0}^{+\infty} a^n z^{-n} = \sum_{n=0}^{+\infty} (a z^{-1})^n$$

$$\boxed{\sum_{k=\alpha}^{\beta} r^k = \frac{r^\alpha - r^{\beta+1}}{1-r}}$$

$$= \frac{1-(a z^{-1})^{+\infty+1}}{1-a z^{-1}} =$$

$|a z^{-1}| < 1 \quad \therefore \quad |z| > |a|$

$$= \frac{1}{1-a z^{-1}}, \quad \text{RoC} \equiv |z| > |a|$$

$$x[n] = -a^n u[-n-1]$$

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$$

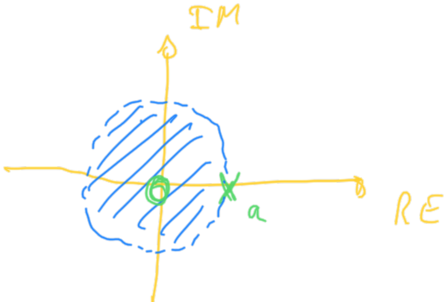
$$= -\sum_{n=-\infty}^{-1} a^n z^{-n} = -\sum_{n=-\infty}^{-1} (a z^{-1})^n$$

$$= -\sum_{m=1}^{+\infty}\left(a z^{-1}\right)^{-m} = -\sum_{m=1}^{+\infty}\left(a^{-1}z\right)^{m}$$

$$= -\frac{a^{-1}z - \left(a^{-1}z\right)^{+\infty+1}}{1 - a^{-1}z} = -\frac{a^{-1}z}{1-a^{-1}z} = \frac{1}{1-az^{-1}}, \quad ROC = |z| < |a|$$

$|a^{-1}z| < 1 \therefore |z| < |a|$

$$X(z) = \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$



——//——

$$X(z) = \frac{z^N - a^N}{z^{N-1}(z-a)}$$

$$z^N = a^N e^{jk2\pi} \qquad \cancel{z = a}$$

$$z = \sqrt[N]{a^N e^{jk2\pi}} = a\, e^{jk\frac{2\pi}{N}}, \quad k = 0, 1, \dots, N-1$$

e.g. $N = 8$