PDSiLab11MAR2021

Lab class # 4

$\boxed{2}$

$$x[n] \xrightarrow{\;Z\;} X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}$$

$$\sum_{n=-\infty}^{+\infty} \boxed{}\; z^{-n}$$

$$(-1)^n x[n] \longleftrightarrow X(-z)$$

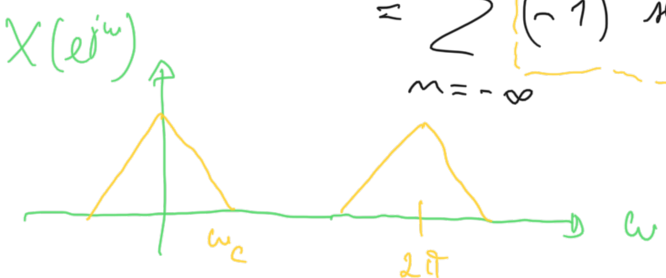
$$\begin{cases} x\left[\frac{n}{2}\right], & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \longleftrightarrow X(z^2)$$

$$x[-n] \longleftrightarrow X(z^{-1})$$

a)

$$\sum_{n=-\infty}^{+\infty} x[n]\,(-z)^{-n} = \sum_{n=-\infty}^{+\infty} x[n]\,\underbrace{(-1)^{-n}}_{=(-1)^n}\, z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} \boxed{(-1)^n x[n]}\; z^{-n}$$



$$X(-z)\Big|_{z=e^{j\omega}} = X(-e^{j\omega}) = X(e^{j\pi}e^{j\omega})$$

$$-1 = e^{\pm j\pi} \qquad = X\left(e^{j(\omega-\pi)}\right)$$

$X(e^{j(\omega-\pi)})$

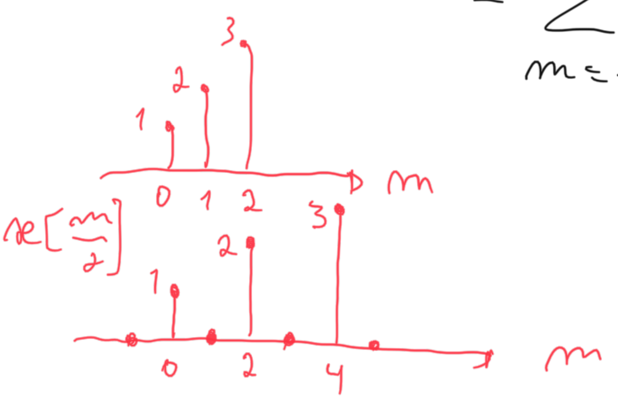
b) $$\sum_{n=-\infty}^{+\infty} x[n]\left(z^{2}\right)^{-n} = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-2n}$$

$2n = m \quad \therefore \quad n = m/2$

$$= \sum_{\substack{m=-\infty \\ m \text{ even}}}^{+\infty} x\left[\frac{m}{2}\right] z^{-m}$$

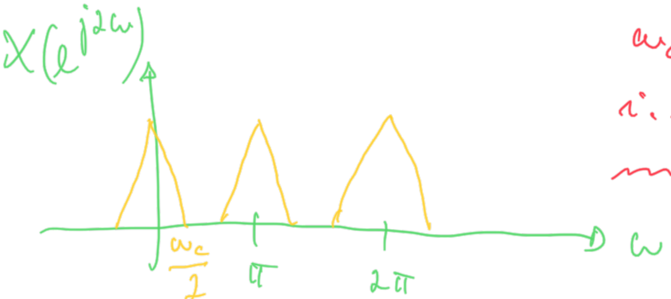
$$= \sum_{m=-\infty}^{+\infty} \boxed{f[m]}\, z^{-m}$$

$x[m]$



$$f[m] = \begin{cases} x\left[\frac{m}{2}\right], & m \text{ even} \\ 0, & m \text{ odd} \end{cases}$$

$$X(z^{2})\Big|_{z=e^{j\omega}} = X(e^{j2\omega})$$



e.g. in this new scale, ω_c appears for $2\omega = \omega_c$ i.e. $\omega = \frac{\omega_c}{2}$, which means the spectrum is compressed by a factor of 2

c) $X(z^{-1})$

$$\sum_{n=-\infty}^{+\infty} x[n]\left(z^{-1}\right)^{-n} = \sum_{n=-\infty}^{+\infty} x[n]\, z^{n}$$

$n = -m$

$$= \sum_{m=-\infty}^{+\infty} x[-m]\, z^{-m}$$

$$X(z^{-1})\Big|_{z=e^{j\omega}} = X(e^{-j\omega})$$

another example:

$$X(z) \longrightarrow X(z^{-3})$$

meaning in words?
what does this mean in the discrete-time domain?
and in the Fourier domain?

$\boxed{3}$

$$H(z) = \frac{2}{1+0.4z^{-1}}, \quad |z| > 0.4$$

$x[n] = u[n]$

a) $$y[n] = \sum_{k=-\infty}^{+\infty} h[k]\, x[n-k]$$

$$H(z) \longleftrightarrow h[n] = 2(-0.4)^{n} u[n]$$

$\boxed{n < 0}$ $\quad y[n] = 0$

$\boxed{n \geq 0}$

$$y[n] = \sum_{k=0}^{n} 2(-0.4)^k = 2\,\frac{1-(-0.4)^{n+1}}{1-\left(-\frac{4}{10}\right)}$$

$$= 1 + \frac{4}{10} = \frac{14}{10}$$

$$= 2\,\frac{10}{14}\left(1-(-0.4)^{n+1}\right)$$

$$y[n] = \frac{10}{7}u[n] - \frac{10}{7}(-0.4)^{n+1}\,u[n]$$

$$= \frac{\cancel{10}}{7}(-0.4)^n\left(-\frac{4}{\cancel{10}}\right)$$

b) $x[n] = u[n] \longleftrightarrow X(z) = \dfrac{1}{1-z^{-1}}, \quad |z| > 1$

$$Y(z) = H(z)\,X(z) = \frac{2}{1+0.4z^{-1}} \cdot \frac{1}{1-z^{-1}}, \quad |z| > 1$$

$$= \frac{A}{1+0.4z^{-1}} + \frac{B}{1-z^{-1}}$$

$$A = \underbrace{(1+0.4z^{-1})}\, Y(z)\Big|_{z=-0.4} = \frac{2}{1-z^{-1}}\Big|_{z=-\frac{4}{10}} = \frac{2}{1+\frac{10}{4}} = \frac{4}{14} =$$

$$B = (1-z^{-1})Y(z)\Big|_{z=1} = \frac{2}{1+\frac{4}{10}} = \frac{20}{14} = \frac{10}{7}$$

$$Y(z) = \frac{4/7}{1+0.4z^{-1}} + \frac{10/7}{1-z^{-1}}$$

$|z|>0.4$ $\qquad$ $|z|>1$

$$y[n] = \frac{4}{7}(-0.4)^n u[n] + \frac{10}{7}u[n]$$

$$Y(z) = \frac{2}{\underbrace{(1+0.4z^{-1})}\underbrace{(1-z^{-1})}} \equiv \frac{\text{"b"}}{\text{"a"}}$$

in Matlab: [1 0.4] [1 -1]

polynomial product is obtained by using

conv([1 0.4], [1 -1])

which produces [1 -0.6 -0.4], this represents

$$1 - 0.6z^{-1} - 0.4z^{-2}$$