PDSI09MAR2021

Lecture # 8

$$x[n] \longleftrightarrow X(z)$$

$$x^*[n] \longleftrightarrow X^*(z^*)$$

$$x[-n] \longleftrightarrow X(1/z)$$

$$x^*[-n] \longleftrightarrow X^*(1/z^*)$$

$$w[n] = x[n] \cdot y^*[n]$$

$$x[n] \longleftrightarrow X(z)$$

$$y^*[n] \longleftrightarrow Y^*(z^*)$$

$$x[n] \cdot y^*[n] \longleftrightarrow \boxed{\frac{1}{2\pi j} \oint_C X(v)\, Y^*\left(\frac{z^*}{v^*}\right) v^{-1}\, dv}$$

$$= w[n] \longleftrightarrow = W(z)$$

$$W(z) \triangleq \sum_{n=-\infty}^{+\infty} w[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} x[n]\, y^*[n]\, z^{-n}$$

$$W(z)\Big| = W(1) = \sum^{+\infty} x[n]\, y^*[n]$$

$$\Big|_{z=1} \qquad \qquad n=-\infty$$

if $y[n] = x[n]$ then

$$W(1) = \sum_{n=-\infty}^{+\infty} x[n]\, x^*[n] = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = E_x$$

$$W(1) = \boxed{\frac{1}{2\pi j} \oint X(v)\, Y^*\!\left(\frac{1}{v^*}\right) v^{-1}\, dv \Big|_{Y(z)=X(z}}$$

$$= \frac{1}{2\pi j} \oint X(v)\, X^*\!\left(\frac{1}{v^*}\right) v^{-1}\, dv$$

$$= \frac{1}{2\pi j} \oint X(z)\, X^*\!\left(\frac{1}{z^*}\right) z^{-1}\, dz = E_x$$

if $z = e^{j\omega}$ $\quad\therefore\quad dz = j e^{j\omega} d\omega$

if $z = e^{j\omega}$ $\quad = \frac{1}{2\pi \cancel{j}} \oint X(e^{j\omega})\, X^*(e^{j\omega}) \frac{1}{\cancel{e^{j\omega}}} \cancel{j}\, \cancel{e^{j\omega}}\, d\omega$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, X^*(e^{j\omega})\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \underbrace{|X(e^{j\omega})|^2}_{\text{spectral density of energy}} d\omega = E$$

$$X(e^{j\omega}) * Y(e^{j\omega}) = \frac{1}{\ } \int^{2\pi} X(e^{j\theta})\, Y(e^{j(\omega-\theta)}) \ldots$$

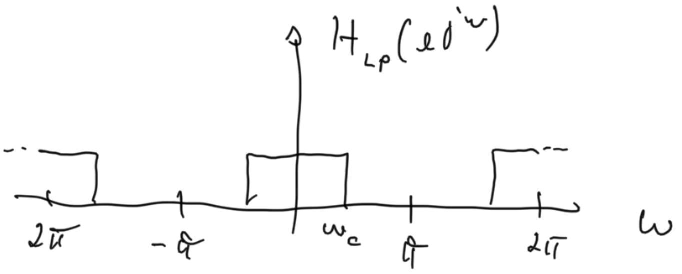
$$X(e^{j\omega}) * Y(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\, Y(e^{j(\omega-\theta)})\, d\theta$$

$$x[n] * y[n] = \sum_{k} x[k]\, y[n-k]$$

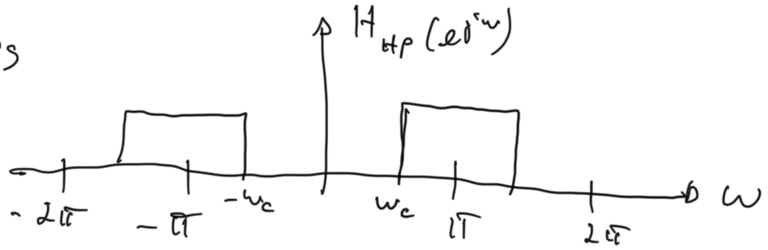
$$X(z) * Y(z) = \frac{1}{2\pi j}\oint X(v)\, Y\left(\frac{z}{v}\right) v^{-1}\, dv$$

IDEAL FILTERS

LOW-PASS

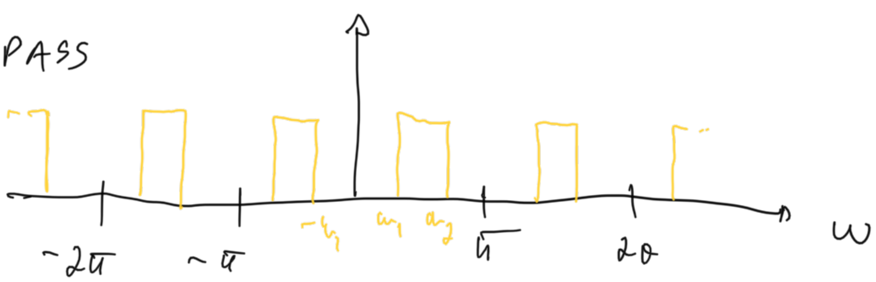


HIGH-PASS



one possibility
$$H_{HP}(e^{j\omega}) = 1 - H_{LP}(e^{j\omega})$$
$$h_{HP}[n] = \delta[n] - h_{LP}[n]$$

BAND-PASS



BAND-STOP

-2π
-π
ω1
ω2
π
2π
ω