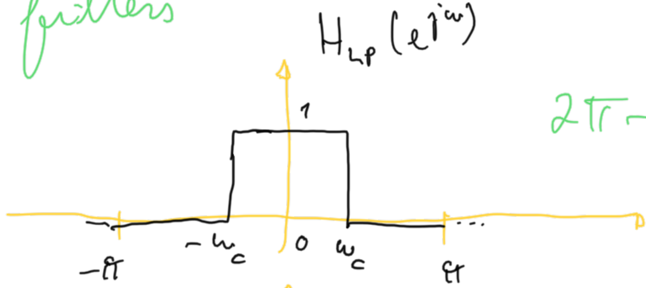
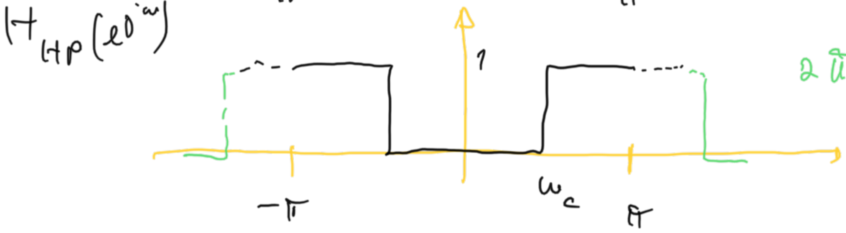
FPS 22 NOV 2021

Lecture

Types of filters

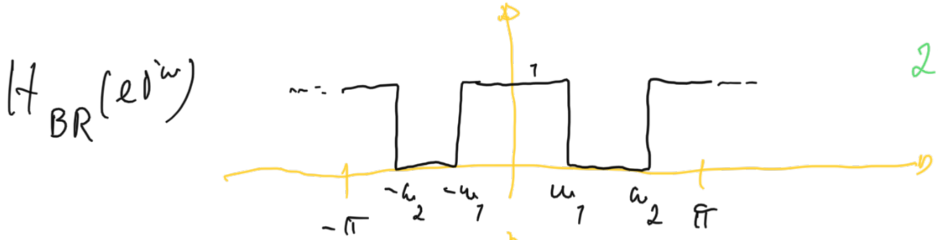


2π – periodic!

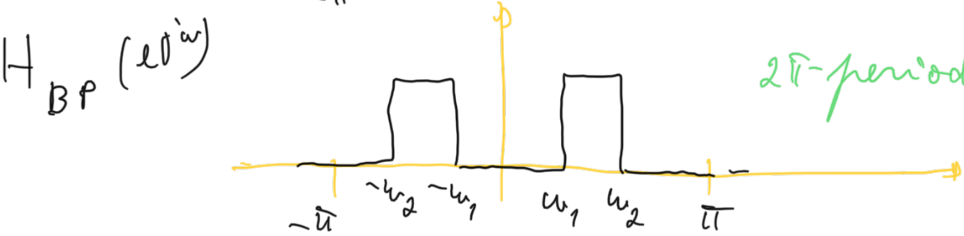


2π – periodic!

$$H_{HP}(e^{j\omega}) = 1 - H_{LP}(e^{j\omega}) \quad \therefore \quad h_{HP}[n] = \delta[n] - h_{LP}[n]$$



2π – periodic!



2π-periodic!

ω

REGARDING PHASE...

what is the ideal phase response of an LTI system/filter?

$$y[n] = H(e^{j\omega_0})\, e^{jn\omega_0}$$

$$H(e^{j\omega_0}) = \mathcal{F}\{h[n]\}\Big|_{\omega=\omega_0} = \left.\sum_{n=-\infty}^{+\infty} h[n]\, e^{-jn\omega}\right|_{\omega=\omega_0}$$

$$x[n] \longrightarrow \boxed{LTI} \longrightarrow y[n]$$

$\sin n\omega_0$ $\qquad |H(e^{j\omega_0})| \sin\left(n\omega_0 + \angle H(e^{j\omega_0})\right)$

$\sin n\omega_1$ $\qquad |H(e^{j\omega_1})| \sin\left(n\omega_1 + \angle H(e^{j\omega_1})\right)$

$$\sin(n - n_D)\omega_0 \longrightarrow \sin(n\omega_0 - n_D\omega_0)$$

$$\sin(n - n_D)\omega_1 \longrightarrow \sin(n\omega_1 - n_D\omega_1)$$

admitting $|H(e^{j\omega_0})| = |H(e^{j\omega_1})| = 1$

in order to keep alignment:

$$\begin{cases} \angle H(e^{j\omega_0}) = -n_D\omega_0 \\ \angle H(e^{j\omega_1}) = -n_D\omega_1 \end{cases}$$

$$\therefore \angle H(e^{j\omega}) = -n_D\,\omega$$

$$\tau(\omega) = -\frac{d}{d\omega} \angle H(e^{j\omega}) \quad \therefore \text{GROUP DELAY}$$

ALL-PASS

$$H(z) = \frac{z - \frac{1}{\alpha^*}}{z - \alpha} \quad \rightarrow x\left(-\frac{1}{\alpha^*}\right)$$

$$H(z) = \frac{1 - \alpha^* z}{z - \alpha} = \boxed{\frac{z^{-1} - \alpha^*}{1 - \alpha z^{-1}}}$$

ALL-PASS!

$|H(e^{j\omega})| = 1$ (condition to be met if it is an all-pass

$$|H(e^{j\omega})|^2 = 1 = H(e^{j\omega})\, H^*(e^{j\omega}) = \underbrace{H(z)\, H^*(1/z^*)\Big|_{z = e^{j\omega}}}_{=1}$$

$$H^*(1/z^*) = \frac{z - \alpha}{1 - \alpha^* z}$$

$$H(z)\, H^*(1/z^*) = \frac{z^{-1} - \alpha^*}{1 - \alpha z^{-1}} \cdot \frac{z - \alpha}{1 - \alpha^* z} = 1$$

$$= \frac{1 - \alpha z^{-1} - \alpha^* z + |\alpha|^2}{1 - \alpha^* z - \alpha z^{-1} + |\alpha|^2} = 1 \quad \therefore \text{ ALL-PASS!}$$