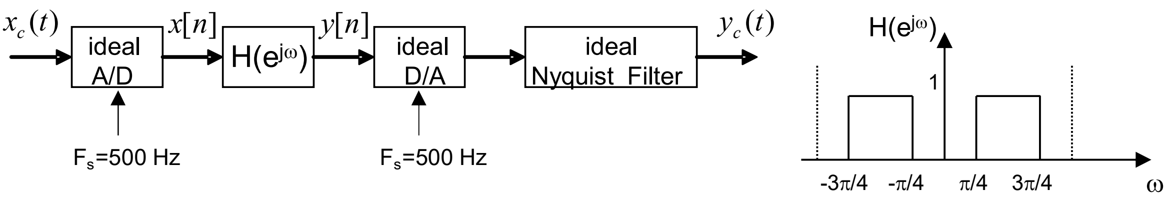
L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2023-2024, week 7
P2P exercises

Exercises related to "Peer-to-peer learning/assessment" (P2P L/A)

P2P Exercise 1

The continuous-time signal $x_c(t) = 1 + \cos(200\pi t) + \sin(700\pi t)$ excites the following system where the sampling frequency is 500 Hz. $H(e^{j\omega})$ represents an ideal band-pass filter whose pass-band is defined in the range $\pi/4 \leq |\omega| \leq 3\pi/4$, as illustrated. Notice that an *anti-aliasing* filter does *not* exist.



a) What are the frequencies, in Hertz, of the analog input signal ?

Note: the answer should be 100 Hz and 350 Hz (which exceed the Nyquist frequency).

b) If an *anti-aliasing* filter existed, what would the equivalent analog system be between $x_c(t)$ and $y_c(t)$ (i.e. from an end-to-end point of view) ?

Note: the answer should be: an analog band-pass filter having cutoff frequencies 62.5 Hz and 187.5 Hz (why ?).

P2P assessment: **1pt /5** if explanation is clear and complete

c) Find the frequencies (in the Nyquist range, i.e. with $-\pi \leq \omega < \pi$) that the discrete-time signal $x[n]$ contains, and write a compact expression for $x[n]$.

Note: the answers should be $\omega_0 = 0$ rad., $\omega_1 = 2\pi/5$ rad., $\omega_2 = -3\pi/5$ rad., and $x[n] = 1 + \cos\left(n\frac{2\pi}{5}\right) - \sin\left(n\frac{3\pi}{5}\right)$.

P2P assessment: **2pt /5** if analytical results are correct and explanation is clear and complete

d) Considering ideal reconstruction, find an expression for $y_c(t)$.

Note: the answer should be $y_c(t) = \cos(200\pi t) - \sin(300\pi t)$.

P2P assessment: **2pt /5** if analytical result is correct and explanation of output frequencies is clear

P2P Exercise 2

Consider the ideal frequency response of the discrete-time filter as specified in P2P exercise 1.

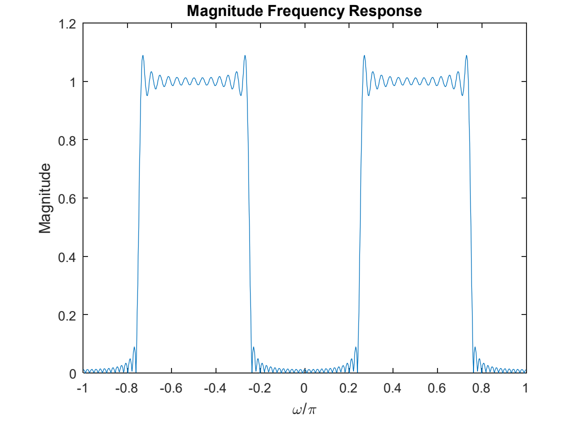
a) Find the ideal impulse response of the filter, $h[n]$.

Note: the answer should be $h[n] = \frac{2}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{4}\right)\right) = \frac{1}{n\pi}\left(\sin\left(\frac{n3\pi}{4}\right) - \sin\left(\frac{n\pi}{4}\right)\right)$

P2P assessment: **3pt /5** if analytical result is correct and explanation is clear and complete

b) Complete the following Matlab code in order to obtain the frequency response that is displayed next

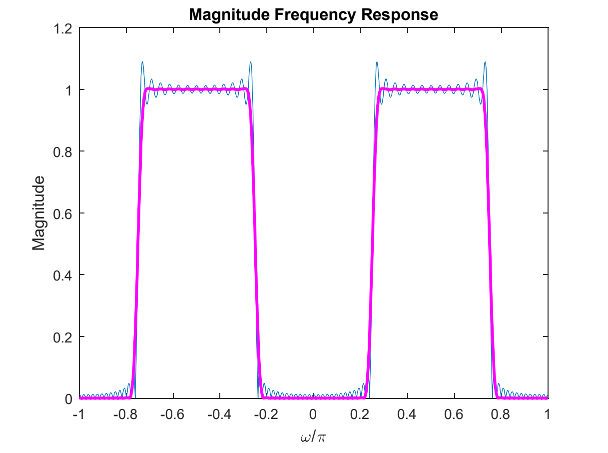
```
n=[-50:50];
wk=[-pi:pi/512:pi-pi/512];
h= TO BE COMPLETED ;
h(51)= TO BE COMPLETED ;
[H W]=freqz(h,1,wk);
plot(W/pi, abs(H))
xlabel('\omega/\pi')
ylabel('Magnitude')
title('Magnitude Frequency Response')
```



P2P assessment: **1pt /5** if Matlab/Octave code correct and obtained figure is as shown

c) Try to explain what the following code does from the point of view of modification of the impulse response in the discrete-time domain, and interpret the corresponding implication in the frequency domain.

```
win=hamming(length(h)).';
hmod=h.*win;
[Hmod W]=freqz(hmod,1,wk);
hold on
plot(W/pi, abs(Hmod),'m')
hold off
```



d) Show how this band-pass filter can be obtained as a transformation of a low-pass filter.

Hint: Consider how the frequency response of a low-pass filter could be used to produce the above frequency response of the band-pass filter.

Note: the answer should be: first, a low-pass filter having $\omega_c = \pi/4$ rad. is designed, then it is modulated to $\omega_M = \pm\pi/2$ rad (the analytical demonstration of this is required).

P2P assessment: **1pt /5** if explanation is clear and if connection to the analytical result of **2a)** is made