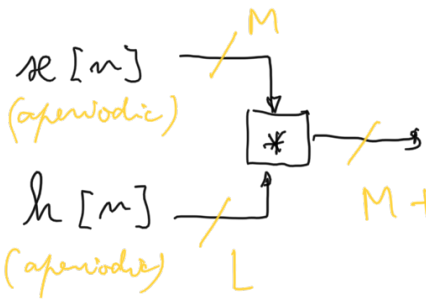
PDSi20APR2021

Lecture # 17

Linear convolution



$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} h[k]\, x[n-k]$$

$M + L - 1 = N$

$$x[n] * h[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega})\, H(e^{j\omega})$$

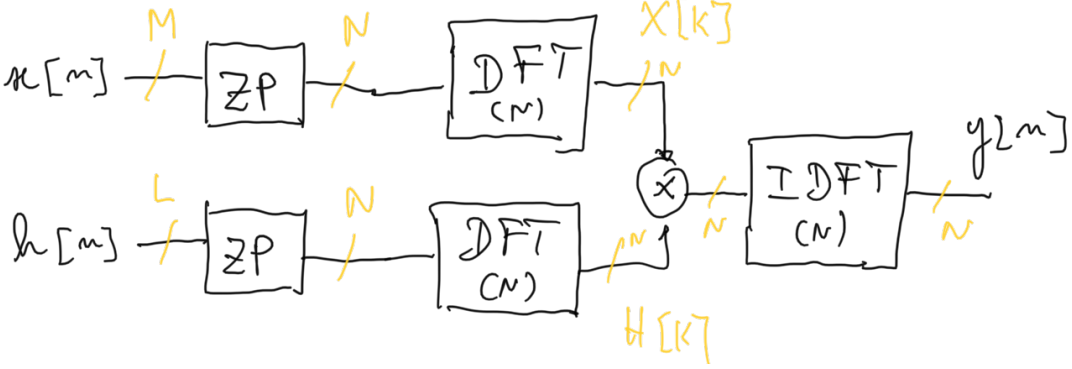
circular convolution

(≡ all sequences are PERIODIC)

DEFINITION

$$x[n] \circledast h[n] \triangleq \sum_{k=0}^{N-1} h[k]\, x[(n-k)_N]$$

all sequences must be N-periodic



$$x[n] \circledast h[n] \xrightarrow{DFT} X[k] \cdot H[k]$$

all: N-periodic (N-periodic)

$$N \geq L + M - 1$$

result of $\circledast$ = result of $*$

D.I.T.

$$X[k] \triangleq \sum_{n=0}^{N-1} x[n] W_N^{kn}, \qquad W_a^b = e^{-j2\pi \frac{b}{a}}$$

$$= \sum_{n=0}^{N/2-1} x[2n] W_N^{k2n} + \sum_{n=0}^{N/2-1} x[2n+1] W_N^{k(2n+1)}$$

$$W_N^{k2n} \quad W_N^{k}$$

$$= \sum_{n=0}^{N/2-1} x[2n] W_N^{k2n} + W_N^{k} \sum_{n=0}^{N/2-1} x[2n+1] W_N^{k2n}$$

$$= W_{N/2}^{kn} \qquad = W_{N/2}^{kn}$$

$$= \sum_{n=0}^{N/2-1} x[2n] W_{N/2}^{kn} + W_N^{k} \sum_{n=0}^{N/2-1} x[2n+1] W_{N/2}^{kn}$$

n=0

n=0