L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2023-2024, week 9
TP (Recitation) exercises

Topics: IIR filter design.

Exercise 1

We want to design a discrete-time, band-pass, IIR filter, with the following specifications: pass-band between 1500 Hz and 3000 Hz, and order 8. The sampling frequency is 22050 Hz. Towards this end, we design four filters: Butterworth, Chebychev type 1, Chebychev type 2, and elliptic. We compare the corresponding frequency response magnitudes, and group delay responses, through the following Matlab code:

```
close all; clear all;
fs=22050; FN=fs/2;
fc=[1500/FN 3000/FN];
[b1 a1]=butter(4, fc); % effective filter order: 2N=8
[H1, W]=freqz(b1, a1);
[grpdly1 W]=grpdelay(b1, a1);
% pass band ripple  = 0.5 dB
[b2 a2]=cheby1(4, 0.5, fc); % effective filter order: 2N=8
[H2, W]=freqz(b2, a2);
[grpdly2 W]=grpdelay(b2, a2);
% stop bands ripple = 25 dB
[b3 a3]=cheby2(4, 25, fc); % effective filter order: 2N=8
[H3, W]=freqz(b3, a3);
[grpdly3 W]=grpdelay(b3, a3);
% pass band ripple = 0.5 dB, stop bands ripple = 25 dB
[b4 a4]=ellip(4, 0.5, 25, fc); % effective filter order: 2N=8
[H4, W]=freqz(b4, a4);
[grpdly4 W]=grpdelay(b4, a4);
subplot(1,2,1);
plot(W/pi*FN/1E3, 20*log10(abs([H1.'; H2.'; H3.'; H4.'])));
xlabel('Frequency (kHz)');
ylabel('Magnitude (dB)');
axis([0 7 -60 5])
subplot(1,2,2);
plot(W/pi*FN/1E3, [grpdly1'; grpdly2'; grpdly3'; grpdly4']);
xlabel('Frequency (kHz)');
ylabel('Group delay (samples)');
axis([0 7 0 90])
```

Based on the analysis of the generated graphical representations, identify which trace (i.e., which colour in the plot) corresponds to which filter. If the choice criterion is narrow transition bands, and moderate group delay distortion (less than 50 samples), which filter would you choose ?

Preliminary consideration

The next problem makes used of the `tf2sos()` Matlab command which is briefly described next (based on the information that is obtained by typing `help tf2sos` on the Matlab command window):

`[SOS,G] = tf2sos(B,A)` finds a matrix SOS in second-order sections form and a gain G which represent the same system H(z) as the one with numerator B, and denominator A, Z-domain polynomials. The poles and zeros of H(z) must be in complex conjugate pairs. SOS is an L by 6 matrix with the following structure:

```
SOS = [ b01   b11   b21  1   a11   a21
        b02   b12   b22  1   a12   a22
        ...
        b0L   b1L  b2L  1   a1L   a2L ]
```

Each row of the SOS matrix describes a 2nd order transfer function:

$$H_k(z) = \frac{b0_k + b1_k z^{-1} + b2_k \; z^{-2}}{1 + a1_k z^{-1} + a2_k \; z^{-2}}$$

where k is the row index. G is a scalar which accounts for the overall gain of the system. If G is not specified, the gain is embedded in the first section.

Exercise 2

Use Matlab's `butter()` command (type `help butter` in Matlab in order to understand how to correctly use this command) to design a 6th-order Butterworth filter, of the 'band stop' type, in order to pass the frequency bands between 0 and $2\pi/5$, and $3\pi/5$ and π, and to reject frequencies in between.

Note: use the `format long` Matlab command to display the '`b`' and '`a`' polynomial coefficients with extended numerical resolution.

a) Add Matlab commands to represent the magnitude of the frequency response on the linear scale (not dB scale). Using the "data cursor" functionality in the Matlab figure, check what the frequency response magnitude is for ω=0 rad, $\omega=2\pi/5$, and $\omega=3\pi/5$. Theoretically, what is the frequency response magnitude for these two frequencies ($\omega=2\pi/5$, $\omega=3\pi/5$) ?

b) Represent the map of the filter's poles and zeros.

c) If the transfer function of our bandstop filter is $H(z)$, it can be looked at the transformation of a filter $F(z)$ in the sense that $H(z) = F(z^2)$. Design in Matlab filter $F(z)$, check its frequency response magnitude, and confirm that the transfer function coefficients of $H(z)$ are related to those of $F(z)$ in agreement with the filter transformation $H(z) = F(z^2)$. How is this transformation manifested in terms of the impulse responses of the so related filters ?

d) For this type IIR system, sketch its direct type II realization structure as well as its transposed version. How many multiplications and additions are required to generate each new output sample ? How many (data, coefficients) memory positions are needed ?

e) In order to implement our bandstop filter, it may be convenient to factorize its transfer function into 2nd-order sub-systems (or biquads). Such a factorization may be obtained using the `tf2sos()` Matlab command. Check what the ouput of this command is. For future reference (regarding the real-time implementation of our bandstop filter on the STM32F7 DSP kit) we may approximate that factorization using:

```
SOS2=[0.5276244  0  0.5276244  1  0  0.50952545
1  0  1  1  0.4763878  0.73873088
1  0  1  1 -0.4763878  0.73873088];
[b2, a2]=sos2tf(SOS2);
[H W]=freqz(b2, a2);
plot(W/pi,abs(H))
```

Exercise 3

Consider the `sting22_e_sinal.wav` audio file that is available on Moodle. This file includes a mixing of a musical excerpt and a natural signal having a repeating pattern. Program a Matlab command `.m` file in order to :

- read the audio file and its parameters,
- design three IIR filters (Butterworth, Chebyshev 1 and Chebyshev 2) of order 8, with a passband between 4600 and 5400 Hz,
- filter the audio file with each one of the designed filters,
- play the filtered audio file.

a) Analyze in Matlab the magnitude and group delay characteristic of the frequency response of each filter.

b) By listening to the signal filtered by each filter, indicate which one is more effective in isolating the natural signal. Relate that effectiveness to the frequency response of the filter.

c) Use Matlab `fdatool`'s filter design environment to repeat the design of the above filters and easily inspect their characteristics (impulse response, frequency response (magnitude, phase and group delay)), as well as their implementation alternatives.