FPS 03 jan2021

Lecture

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$

$$W_a^b = e^{-j2\pi \frac{b}{a}}$$

$$W_N^{kn} = e^{-j\frac{2\pi}{N} kn}$$

$$= \sum_{n=0}^{N/2-1} x[2n]\, W_N^{k2n} + \sum_{n=0}^{N/2-1} x[2n+1]\, W_N^{k(2n+1)}$$

$$W_N^{k2n} = e^{-jk\frac{2\pi}{N}2n} = e^{-jk\frac{2\pi}{N/2}n} = W_{N/2}^{kn}$$

$$W_N^{k(2n+1)} = W_N^{k2n} W_N^{k}$$

$$= \sum_{n=0}^{N/2-1} x[2n]\, W_{N/2}^{kn} + W_N^k \sum_{n=0}^{N/2-1} x[2n+1]\, W_{N/2}^{kn}$$

$G[k]$: $\frac{N}{2}$-periodic

$H[k]$: $\frac{N}{2}$-periodic

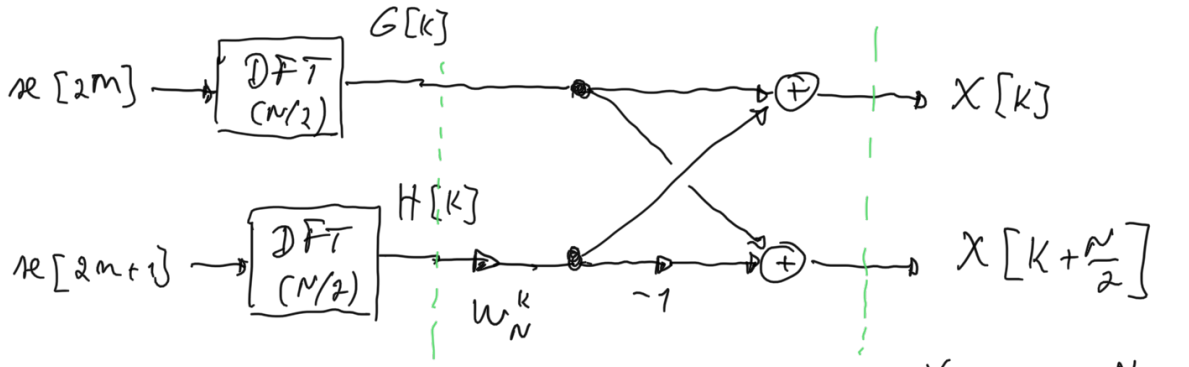
$$= G[k] + W_N^k H[k], \quad k = 0, 1, \dots, N-1$$

$$X[k+\tfrac{N}{2}] = G[k+\tfrac{N}{2}] + W_N^{(k+\frac{N}{2})} H[k+\tfrac{N}{2}]$$

$$= G[k] + W_N^k W_N^{N/2} H[k]$$

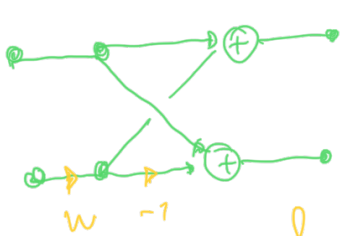
$$W_N^{N/2} = e^{-j\frac{2\pi}{N}\frac{N}{2}} = e^{-j\pi} = -1$$

$$= G[k] - W_N^k H[k], \quad k = 0, 1, \dots, \tfrac{N}{2}-1$$

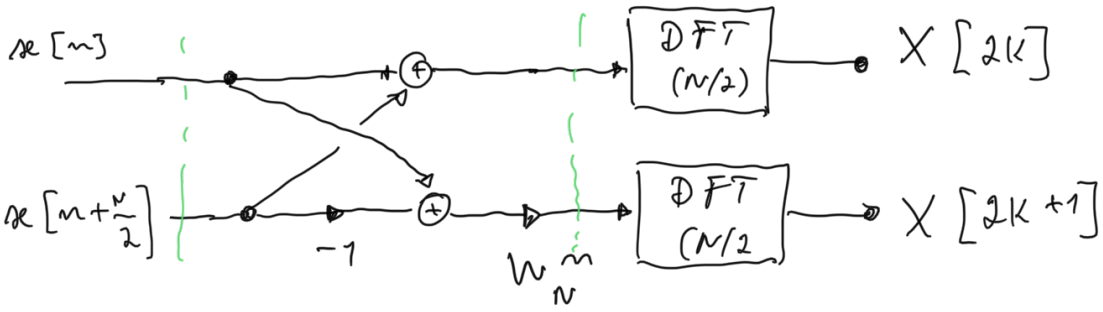


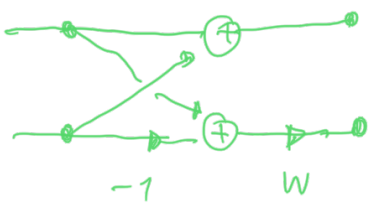
$K = 0, 1, \dots, \frac{N}{2} - 1$

gain : $\dfrac{N^2}{\left(\frac{N}{2}\right)^2 \times 2 + \frac{N}{2}} = \dfrac{N^2}{\frac{N^2}{2} + \frac{N}{2}} = \dfrac{N^{\cancel{2}}}{\cancel{\frac{N}{2}}\left(1 + N\right)} = \dfrac{2}{1 + \frac{1}{N}}$



butterfly D.I.T.





butterfly D.I.F.