PDSi22APR2021

Lecture # 18

$$X[k] \triangleq \sum_{n=0}^{N-1} x[n]\, W_N^{kn} \qquad O(N^2)$$

DiT

(from previous class:)

$$X[k] = \underbrace{\sum_{n=0}^{N/2-1} x[2n]\, W_{N/2}^{kn}}_{G[k]\ :\ N/2\text{-periodic}} + W_N^k \underbrace{\sum_{n=0}^{N/2-1} x[2n+1]\, W_{N/2}^{kn}}_{H[k]\ :\ \frac{N}{2}\text{-periodic}}$$

$$X[k] = G[k] + W_N^k\, H[k]$$

$k = 0, 1, \dots, N-1$

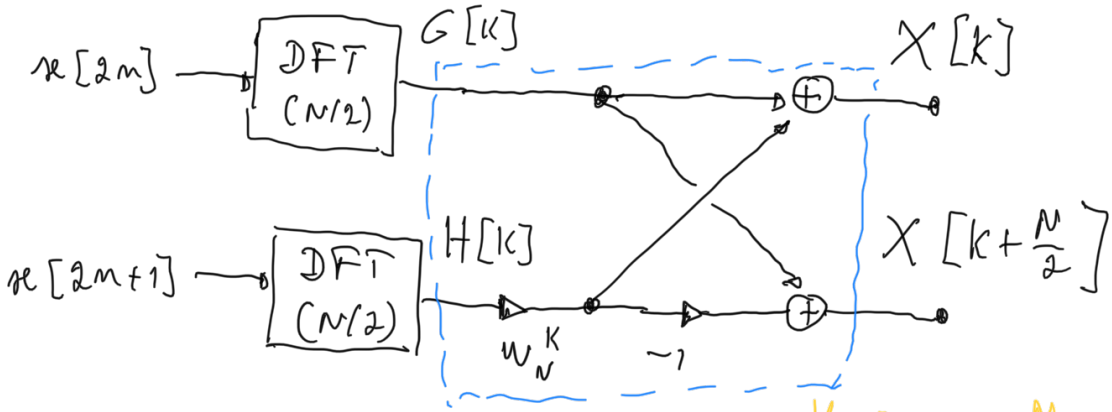
$$X[k+\tfrac{N}{2}] = G[k+\tfrac{N}{2}] + W_N^{k+\frac{N}{2}}\, H[k+\tfrac{N}{2}]$$

$G[k]$ $\qquad$ $H[k]$

$$= W_N^k \underbrace{W_N^{N/2}}_{= e^{-j2\pi \frac{N/2}{N}} = e^{-j\pi} = -1}$$

$$\begin{cases} X[k+\frac{N}{2}] = G[k] - W_N^k\, H[k] \\ X[k] = G[k] + W_N^k\, H[k] \end{cases}$$

$k = 0, 1, \dots, \frac{N}{2}-1$



$k = 0, 1, \dots, \frac{N}{2} - 1$

$\Rightarrow$ butterfly

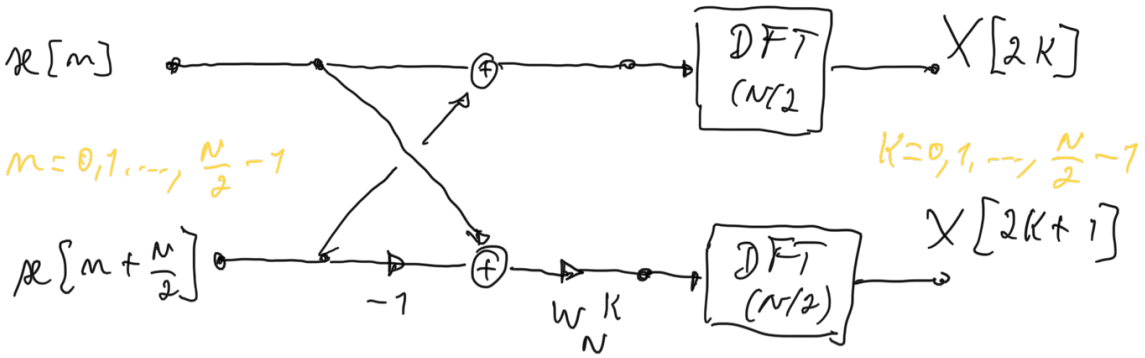
computational "cost":

$$2\left(\frac{N}{2}\right)^2 + \frac{N}{2} = \frac{N^2}{2} + \frac{N}{2} = \frac{N}{2}(N+1)$$

computational "advantage"?

$$\frac{N^2}{\frac{N}{2}(N+1)} = \frac{2}{1+\frac{1}{N}} \dashrightarrow \sim 2$$

DECIMATION IN FREQUENCY (DiF)



$n = 0, 1, \dots, \frac{N}{2} - 1$

$k = 0, 1, \dots, \frac{N}{2} - 1$

$\therefore$ DiF is the transposed structure of DiT