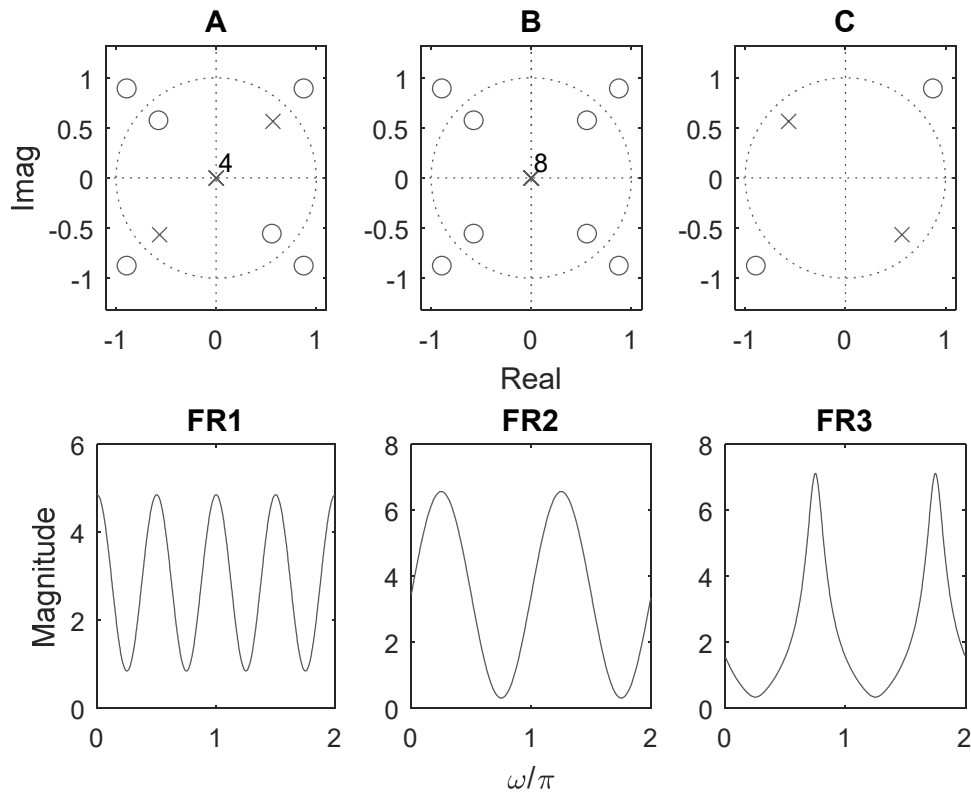
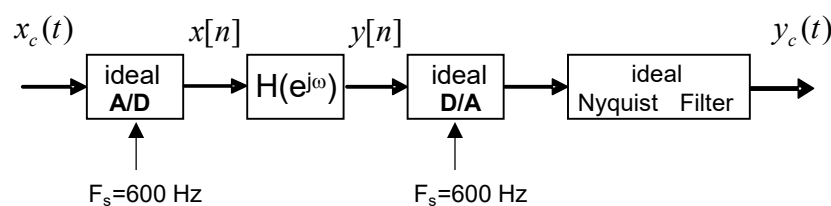


NOTE: each question *must* be answered in a separate sheet; please provide complete answers

1. The zero-pole diagrams of three different causal discrete-time systems (A, B, C) are illustrated next, as well as their frequency response magnitudes (FR1, FR2, FR3).



- a) [2 pts] Match each zero-pole diagram (A, B, C) to the corresponding frequency response magnitude (FR1, FR2, FR3), and present the main supporting arguments.
- b) [1,5 pts] For each zero-pole diagram (A, B, C), indicate if the corresponding system is linear-phase and/or if its impulse response is real-valued. Justify.
- c) [1 pt] If the impulse responses of systems A, B, and C are represented by $h_A[n]$, $h_B[n]$ and $h_C[n]$, respectively, explain if some real-valued constant $\theta \neq 1$ exists such that making $\theta^n h_A[n]$, $\theta^n h_B[n]$, and $\theta^n h_C[n]$, does not modify the original systems.
2. The illustrated signal processing chain includes a discrete-time system that is governed by the difference equation $y[n] = x[n] - x[n - 2] + x[n - 4]$. The sampling frequency is 600 Hz and the analog input is $x_c(t) = 1 + \sin(100\pi t) + \sin(1050\pi t)$. Notice that an *anti-aliasing* filter does not exist.



- a) [1 pt] Find the frequencies of the discrete-time signal $x[n]$ in the Nyquist range, i.e. in the range $-\pi \leq \omega < \pi$. Obtain a compact expression for $x[n]$.

- b) [2 pts] Obtain a compact expression for the frequency response of the discrete-time system and sketch its magnitude and phase responses.
- c) [1,5 pts] Obtain $y[n]$ and, presuming ideal reconstruction conditions, obtain $y_c(t)$.
3. In one of the FPS Labs, the following code was used to service an interrupt-based routine whose relevant C code is as follows (assume that $w[]$ represents a vector of floating point numbers that is defined and initialized outside the scope of this routine):

```
int16_t i;
float32_t w0, yn;
// r is a constant that is defined externally

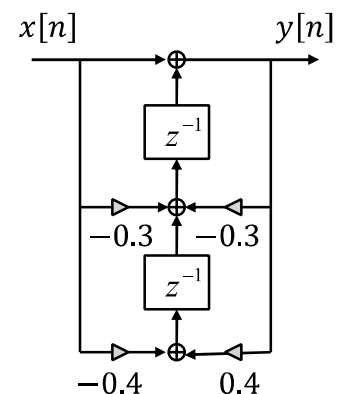
w0 = (float32_t)(rx_sample_L);

w0 += ( (float32_t)(r) * w[1] - (float32_t)(r*r) * w[2] );
yn = (float32_t)(r*r) * w0 - (float32_t)(r) * w[1] + w[2];
tx_sample_L = (int16_t)(yn);

w[0] = w0;
for (i=2 ; i>0 ; i--) w[i] = w[i-1];
return;
```

- a) [0,5 pts] Explain what is the length of vector $w[]$ and what is the order of the discrete-time system that this C code implements.
- b) [1,5 pts] Sketch the realization structure of the discrete-time system that this C code implements and write its transfer function (including the RoC).
4. The realization structure of a causal discrete-time system is depicted next.

- a) [1 pt] Obtain the difference equation that the illustrated realization structure implements and find the corresponding transfer function (including the RoC).
- b) [1,5 pts] Find the zeros and poles of the discrete-time system and represent them on the Z plane.
- c) [1 pt] Justify which of the following sentences is true: the inverse system is obtained by: i) changing the sign of *just* one coefficient in the realization structure, ii) changing *just* one coefficient in the realization structure, iii) changing more than one coefficient in the realization structure.

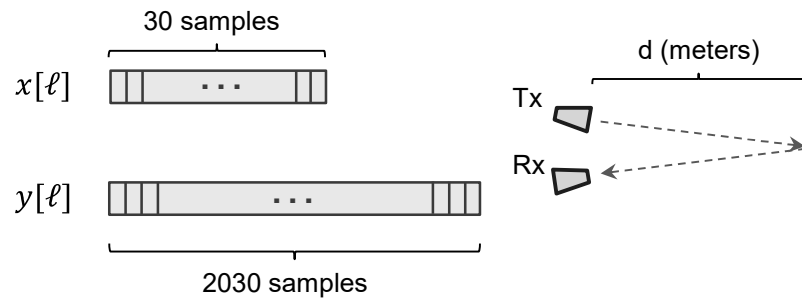


5. Consider the following Matlab code.

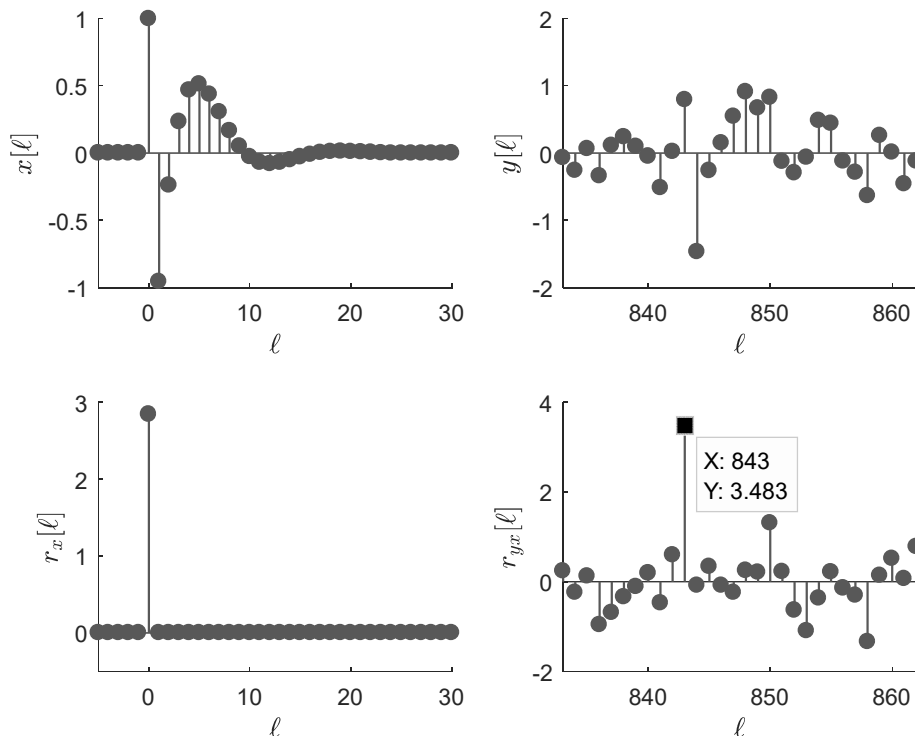
```
x=[1j 2j 3j 4j]; N=length(x);
X=fft(x);
Y=real(j*conj(X));
ifft(Y)
W=X; W(2:N)=X(N:-1:2);
ifft(W.*Y)
```

- a) [1,5 pts] Find and explain the result of `ifft(Y)` without computing any FFT/IFFT.
- b) [2 pts] Find and explain the result of `ifft(W.*Y)` without computing any FFT/IFFT.
- Note:** You may assume here that `ifft(Y) = [1 3 3 3]`.

6. A simplified SONAR system is illustrated next that transmits (Tx) an acoustic 30-sample probe signal, $x[\ell]$. After hitting a target at distance d (in meters) from the transmission point, the echo of that probe travels back and is captured by a receiver (Rx) that is co-located with the transmitter, and is stored in vector $y[\ell]$ whose length is 2030 samples. The sampling frequency is 22050 Hz and we admit that the speed of sound is 340 m/s. To simplify, we admit that the receiver starts listening to the echo of the probe when the transmitter starts transmitting the probe, and we admit further that both probe and echo travel horizontally (and not obliquely).



The following figure represents the probe signal, $x[\ell]$, and its auto-correlation function $r_x[\ell]$, which consists of an impulse. In one experiment, when the transmitter starts transmitting $x[\ell]$, the receiver starts collecting 2030 samples of the noisy signal $y[\ell]$, a relevant region of which is illustrated next. The cross-correlation $r_{yx}[\ell]$, between $y[\ell]$ and $x[\ell]$, is computed and a relevant peak is detected, as also illustrated next.



- a) [1 pt] What is the maximum distance this basic SONAR is able to detect ? Justify.
b) [1 pt] Using the provided information, what is the distance of the detected target ? Justify.