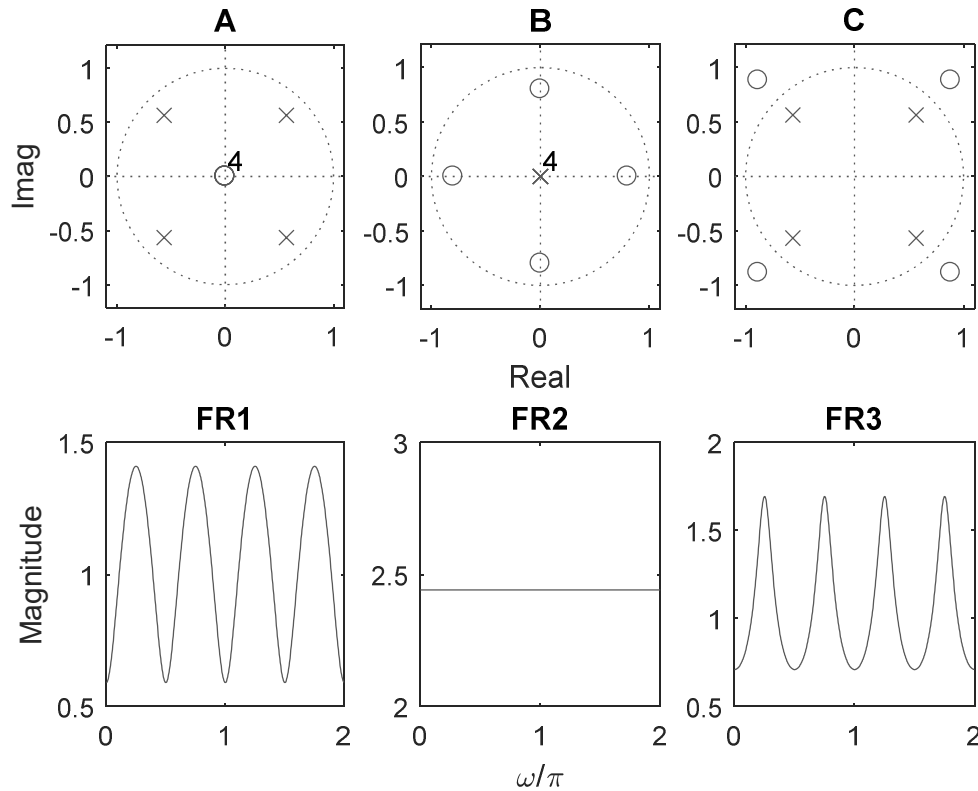
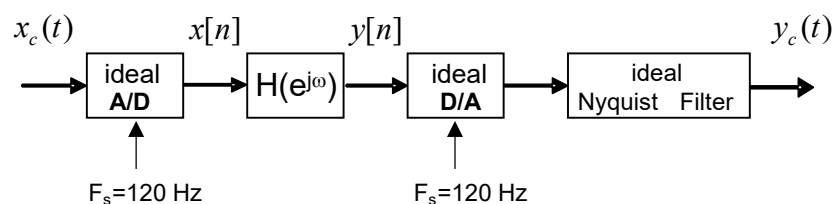


NOTE: each question *must* be answered in a separate sheet; please write your name and Student order number on all sheets, please provide complete answers while trying to minimize paper usage.

1. Three causal discrete-time systems have the illustrated zero-pole diagrams A, B, and C, and the illustrated frequency response magnitudes FR1, FR2, and FR3.



- a) [2 pts] Match each zero-pole diagram (A, B, C) to the corresponding frequency response magnitude (FR1, FR2, FR3), and present the main supporting arguments.
- b) [1 pt] For each zero-pole diagram (A, B, C), indicate if the corresponding system is FIR or IIR, and whether or not it is minimum-phase. Justify.
- c) [1 pt] If the transfer functions of systems A, B, and C are represented by $H_A(z)$, $H_B(z)$ and $H_C(z)$, respectively, explain if $H_C(z)$ may be expressed as a combination of the remaining two.
2. The illustrated signal processing chain includes a discrete-time system that is governed by the difference equation $y[n] = x[n] - x[n - 3]$. The sampling frequency is 120 Hz and the analog input is $x_c(t) = 1 + \sin(80\pi t) + \sin(540\pi t)$. Notice that an *anti-aliasing* filter does not exist.



- a) [1,5 pts] Find the frequencies of the discrete-time signal $x[n]$ in the Nyquist range, i.e. in the range $-\pi \leq \omega < \pi$. Obtain a compact expression for $x[n]$.
 - b) [2 pts] Obtain a compact expression for the frequency response of the discrete-time system, and sketch its magnitude and phase responses.
 - c) [1,5 pts] Obtain $y[n]$ and, presuming ideal reconstruction conditions, obtain $y_c(t)$.
3. In one of the FPS Labs, the following code was used to service an interrupt-based routine whose most relevant C code is as follows (assume that N and $BETA$ are constants defined outside the scope of this routine, and $w[]$ represents a vector of $N+1$ floating point numbers initialized to zero outside the scope of this routine):

```
int16_t i;
float32_t w0, yn;

w0 = (float32_t)(rx_sample_L);

w0 += (float32_t)(BETA) * w[N];
yn = w0;
tx_sample_L = (int16_t)(yn);

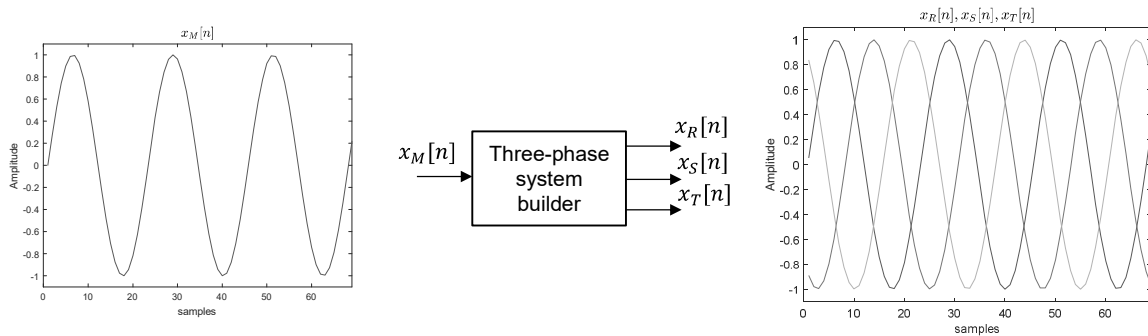
w[0] = w0;
for (i=N ; i>0 ; i--) w[i] = w[i-1];
return;
```

- a) [1 pt] Explain with words the operation of this code and write the difference equation it implements.
 - b) [1,5 pts] Sketch the realization structure of the discrete-time system that is implemented by this code and write its transfer function (including the RoC).
4. Consider the following Matlab code.

```
x=[3 2 1 -1j -2j -3j]; N=length(x);
X=fft(x);
Y(1)=conj(X(1));
Y(2:N)=conj(X(N:-1:2));
Xe=(X+Y)/2; Xo=(X-Y)/2;
z=ifft(Xe.*(1j*Xo))
```

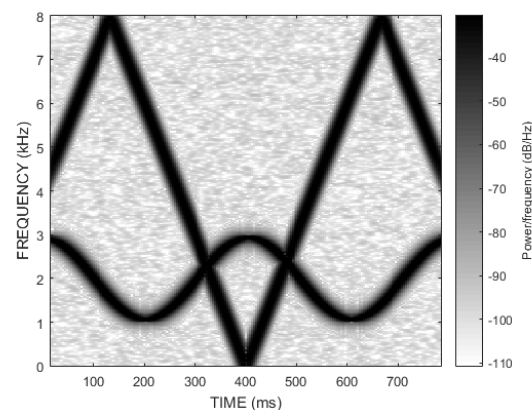
- a) [0,5 pts] Without executing the code, find and explain the result of $\text{ifft}(1j * X_o)$.
 - b) [1,5 pts] Without executing the code, find and explain the contents of vector z .
5. In one of the TP classes, the Hilbert Transformer was studied as a useful phase shifter. Here, we generalize that concept to an arbitrary phase shifter whose ideal frequency response is $H(e^{j\omega}) = \begin{cases} e^{j\theta}, & -\pi \leq \omega < 0 \\ e^{-j\theta}, & 0 \leq \omega < \pi \end{cases}$, where θ is the arbitrary phase.
- a) [1,5 pts] Find the ideal impulse response of such an arbitrary phase shifter. Specify that response for $n = 0$, and for $n \neq 0$.

- b) [1,5 pts] Admit that you want to build a three-phase system ($x_R[n]$, $x_S[n]$, $x_T[n]$) from a single-phase (monophasic) system ($x_M[n] = \sin(n\omega_M)$), as illustrated next, by using FIR versions of the above arbitrary phase filter. Using a schematic representation, explain how you could achieve that, and explain if your solution would work both for the European power grid frequency (50 Hz), and the North-American power grid frequency (60 Hz).



6. Consider the illustrated spectrogram in the *Nyquist* range. Admit that the acquisition system does not include an *anti-aliasing* filter. Admit further that two real-valued signals are being analyzed, that their starting frequencies (for $t=0.0$ s) are within the Nyquist range, and that they evolve *monotonously* with time.

Note: Darker colors mean higher Power Spectral Densities



- a) [1 pt] Describe briefly each one of the two represented signals.
b) [1 pt] Represent a plausible periodogram (or power spectrum) corresponding to $t=0.1$ s and corresponding to $t=0.5$ s. Explain.
c) [1,5 pts] Now, admit that an *anti-aliasing* filter exists in the system. Sketch the spectrogram that would result from this new situation and highlight the differences relative to the spectrogram illustrated above.