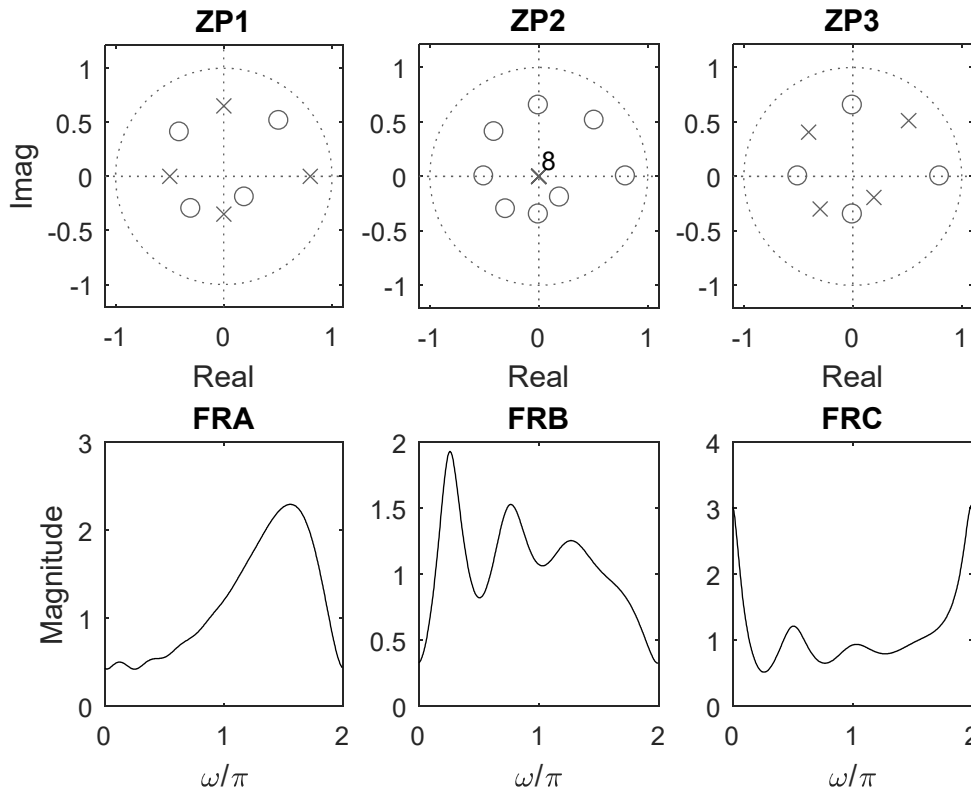


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1. Three causal discrete-time systems have their zero-pole diagrams (ZP1, ZP2, ZP3), and magnitude frequency responses (FRA, FRB, FRC), as represented next.



- a) [2 pts] Indicate if the following sentences are true, or false, and, in each case, briefly explain why:

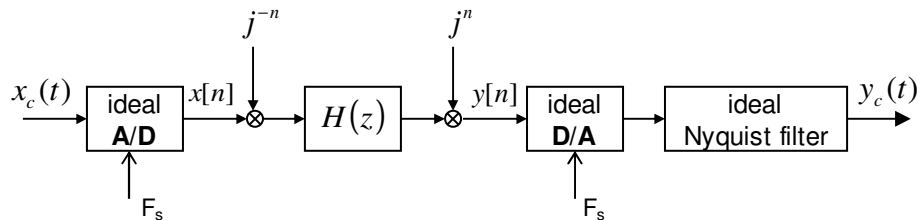
- «The order of all three systems is 8»
- «All systems are minimum-phase»
- «All systems have a real-valued impulse response»
- «If two of the above systems are cascaded, a zero-order system results»
- «if two of the above systems are cascaded, a eight-order system results»

- b) [1.5 pts] For each system, make the most plausible correspondence between zero-pole diagram and frequency response. Briefly explain your reasoning.

2. Consider the illustrated system where  $H(z)$  represents an ideal discrete-time low-pass filter whose specification is  $H(e^{j\omega})=1$ ,  $|\omega| < \pi/4$ , and zero otherwise. The

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analog input to the system is  $x_c(t) = e^{j420\pi t} + \sin(500\pi t/3) + \cos(800\pi t/3)$ . Assume ideal A/D and D/A conversion as well as ideal reconstruction. The sampling frequency is  $F_s=100$  Hz. Notice that an anti-aliasing filter does not exist.



- [1 pt]** Find a simplified expression for  $x[n]$  such that its frequencies are in the range  $[-\pi, \pi]$  rad.
  - [2 pts]** Obtain the relationship between  $Y(e^{j\omega})$  and  $X(e^{j\omega})$ , as a function of  $H(e^{j\omega})$ , and sketch it in one period in  $\omega$ .
  - [1 pt]** Obtain  $y[n]$  and, considering ideal reconstruction, obtain  $y_c(t)$ .
3. Consider that  $x[n] \xrightarrow{DFT} X[k]$ ,  $n, k = 0, 1, \dots, N-1$ , and that a  $2N$ -periodic vector  $Y[k]$  is created as  $Y[k] = Y[N+k] = X[k]$ ,  $k = 0, 1, \dots, N-1$ .
- [1 pt]** Express  $y[n]$  as a function of  $x[n]$ .
  - [1 pt]** If a new  $2N$ -periodic vector  $f[n]$  is created as  $f[n] = y[n] + y[(n-1)_{2N}]$ ,  $n = 0, 1, \dots, 2N-1$ , express  $F[k]$  as a function of  $Y[k]$ .

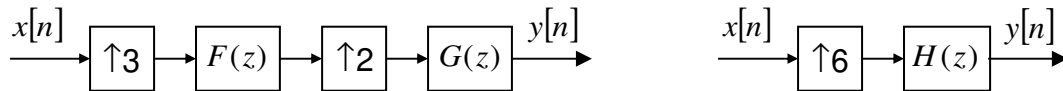
Now consider the following Matlab code.

```
x=[1 2 3 4 5]; N=length(x);
X=fft(x); Y=[X X]; ifft(Y)
F=Y.*(1+exp(-j*pi/N*[0:2*N-1])); ifft(F)
ifft(j*imag(X).*real(X))
```

- [0.5 pts]** Using a) and without computing any FFT or IFFT find the output of `ifft(Y)`.
- [0.5 pts]** Using b) and without computing any FFT or IFFT find the output of `ifft(F)`.
- [2 pts]** Without computing any FFT or IFFT find the output of `ifft(j*imag(X).*real(X))`.

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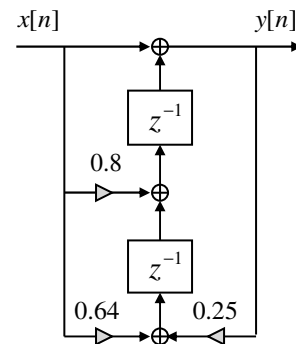
4. [2.5 pts] Consider the following two equivalent multirate systems.



Express  $H(z)$  as a function of  $F(z)$  and  $G(z)$ . Indicate one advantage and one inconvenient of the system on the left relatively to the system on the right.

5. The realization structure of a causal discrete-time system is depicted next.

- [1 pt] Obtain a difference equation describing the system as well as its transfer function  $H(z)$ .
- [1 pt] Find the zeros and poles of the system and represent them in the Z plane.
- [1 pt] Sketch a direct type-2 (canonic) realization structure of the inverse system.



6. [2 pt] Two methods were studied in PDSi that permit to design an IIR discrete-time filter from an existing analog filter. Describe the main ideas underlying each one of those two methods and discuss their relative advantages and disadvantages.

END