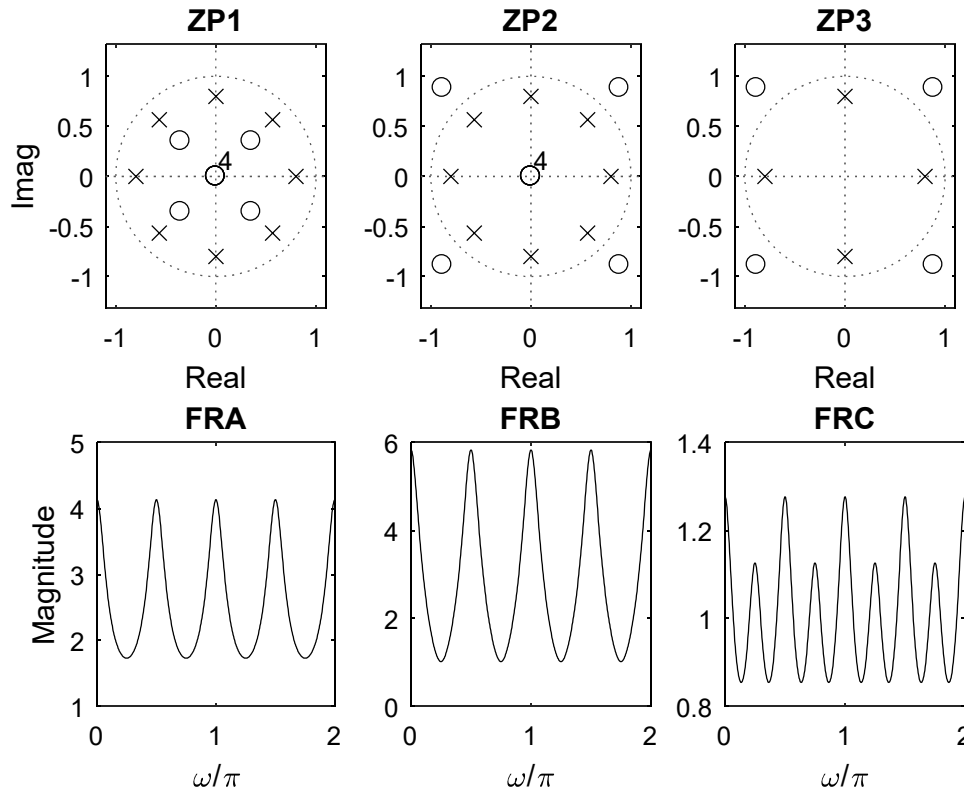


Closed book exam, except for the provided formulae sheet. Duration: 90m.

1. Three causal discrete-time systems have their zero-pole diagrams (ZP1, ZP2, ZP3), and magnitude frequency responses (FRA, FRB, FRC), as represented next.

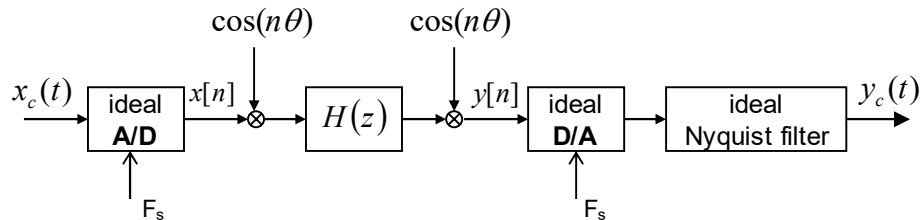


- a) [2 pts] Indicate if the following sentences are true, or false, and briefly explain why:
- i) «All systems have a real-valued impulse response»
 - ii) «Two systems are maximum-phase»
 - iii) «Only one system has order 8»
 - iv) «The order of one system can be reduced from 8 to 4 without modifying its magnitude frequency response»
 - v) «One system is a minimum-phase version of another system»
- b) [1.5 pts] For each system, make the most plausible correspondence between zero-pole diagram and frequency response. Briefly explain your reasoning.

2. Consider the illustrated system where $H(z)$ represents an ideal discrete-time low-pass filter whose specification is $H(e^{j\omega}) = 1$, $|\omega| < \pi/4$, and zero otherwise. The

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analog input to the system is $x_c(t) = e^{j650\pi t/3} + e^{j400\pi t/3} + e^{j450\pi t}$. Assume ideal A/D and D/A conversion as well as ideal reconstruction. The sampling frequency is $F_s=100$ Hz. Notice that an anti-aliasing filter does not exist.



- a) [1 pt] Find a simplified expression for $x[n]$ such that its frequencies are in the range $[-\pi, \pi]$ rad.
 - b) [2 pts] Express $Y(z)$ as a function of $X(z)$ and $H(z)$, and show that when $\theta = \pi/2$, that relationship reduces to $Y(z) = \frac{1}{4}[X(z) + X(-z)][H(jz) + H(-jz)]$.
 - c) [1 pt] Assuming that $\theta = \pi/2$, obtain $y[n]$ and, admitting ideal reconstruction, obtain $y_c(t)$.
3. Consider that $x[n] \xrightarrow{DFT} X[k]$, $n, k = 0, 1, \dots, N-1$, and that a $2N$ -periodic vector $Y[k]$ is created as $Y[2k] = 2X[k]$, $k = 0, 1, \dots, N-1$, and $Y[2k+1] = 0$, $k = 0, 1, \dots, N-1$.

- a) [1 pt] Express $y[n]$ as a function of $x[n]$.
- b) [1 pt] If a new $2N$ -periodic vector $F[k]$ is created as $F[2k] = F[2k+1] = 2X[k]$, $k = 0, 1, \dots, N-1$, express $f[n]$ as a function of $x[n]$.

Now consider the following Matlab code.

```
x=[5 4 3 2 1]; X=fft(x); N=length(x);
Y=zeros(1, 2*N); Y(1:2:end)=2*X; y=ifft(Y)
F=zeros(1, 2*N); F(1:2:end)=2*X; F(2:2:end)=2*X;
f=y.*(1+exp(j*pi/N*[0:2*N-1])); stem(f-ifft(F))
ifft(cconv(X,X,N))/N
```

- c) [1 pt] Using a) and without computing any FFT or IFFT find the output of $\text{ifft}(Y)$.

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- d) [1 pt] Using b) and without computing any FFT or IFFT describe the output of the `stem(f-iff(F))` command.
- e) [1 pt] Without computing any FFT or IFFT find the output of `iff(cconv(X,X,N))/N`. where `cconv(A,B,N)` represents the modulo-N circular convolution between vectors A and B.

4. [2.5 pts] Consider the following two multirate systems. Notice that no filters exist.



In each case, express $Y(z)$ as a function of $X(z)$. Indicate under which conditions the two systems are equivalent.

5. The zeros of a second-order causal discrete-time system are $z = \pm 1.25$, and the poles are $z = 0.8e^{\pm j\pi/3}$. The gain of the system for $\omega = 0$ is 1.
- a) [1 pt] Obtain its transfer function, $H(z)$, as well as its difference equation.
- b) [1 pt] Sketch a canonic realization structure of the system and indicate what its computational cost is in terms of memory and arithmetic operations.
- c) [1 pt] Obtain a minimum-phase version of the system that keeps the gain of the magnitude frequency response of original system.
6. [2 pts] Two methods were studied in PDSi that permit to design linear-phase discrete-time filters. Describe the main ideas underlying each one of those two methods and discuss their relative advantages and disadvantages.

END