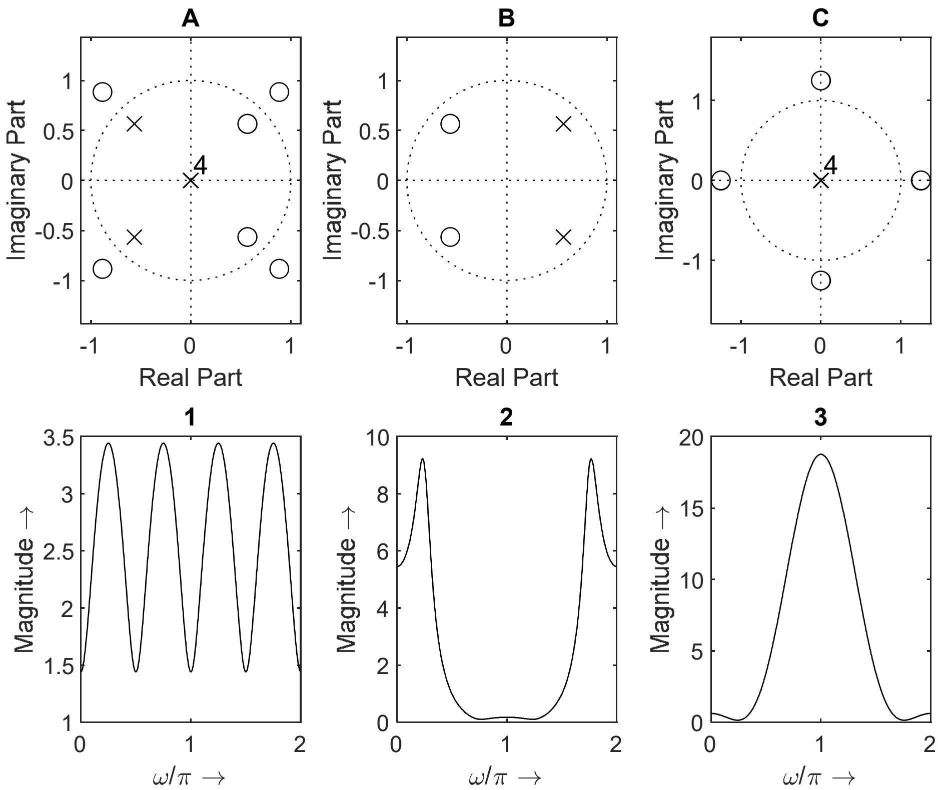
L.EEC/M.EEC
L.EEC025 - Fundamentals of Signal Processing

FIRST EXAM, FEBRUARY 01, 2024
Duration: 120 Minutes, closed book

NOTE: each question ***must*** be answered in a separate sheet; please write your name and Student order number on all sheets, please provide complete answers while trying to minimize paper usage.

1. Three causal discrete-time systems have the illustrated zero-pole diagrams A, B, and C, and the illustrated frequency response magnitudes 1, 2, and 3. Admit that the radius of all poles and zeros is either 0.8 or 1/0.8, and that the angles of all poles and zeros are multiples of $\pi/4$ rad.



a) **[1,5 *pts*]** Match each zero-pole diagram (A, B, C) to the corresponding frequency response magnitude (1, 2, 3), and indicate the main supporting arguments.

b) **[1 *pt*]** Which of the represented systems (A, B, or C) are stable ? And which ones are minimum-phase systems ? Why ?

c) **[1 *pt*]** Consider the new system formed by cascading systems A and B. What is the order of the new system ? Is the new system IIR or FIR ? Is it a linear-phase system ?

d) **[1 *pt*]** «If $H_A(z)$, $H_B(z)$, and $H_C(z)$ represent the transfer function of systems A, B and C, respectively, then $H_A(z)$ may be obtained by combining $H_B(z)$ and $H_C(z)$ in a suitable way». Is this statement true or false? If true, what is the suitable combination?

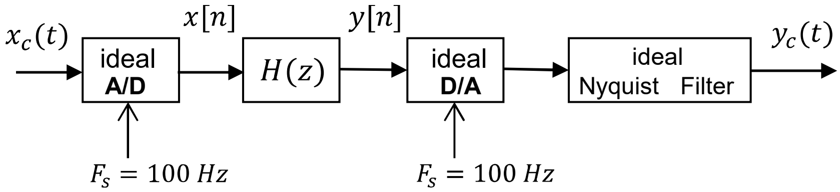
2. Consider system C as described in Prob. **1**.

a) **[1,5 *pts*]** Find the transfer function of the system, $H(z)$, write a difference equation implementing it and sketch a corresponding canonic realization structure.

(***continues***)

b) **[1,5 *pt*]** Obtain a compact expression characterizing the magnitude of the frequency response of the system, $\left|H\left(e^{j\omega}\right)\right|$, and show that its maximum gain depends on $\sqrt{1+2\times 0.8^{-4}+0.8^{-8}}$, and its minimum gain depends on $\sqrt{1-2\times 0.8^{-4}+0.8^{-8}}$.

c) **[1 *pt*]** Consider the illustrated analog and causal discrete-time system whose frequency response is as suggested in **b)**. The sampling frequency is 100 Hz. The analog input signal is $x_c(t)=\sin(175\pi t)+\sin(450\pi t)$. Notice that an *anti-aliasing* filter does not exist.



Find the sinusoidal frequencies of the discrete-time signal $x[n]$ in the Nyquist range, i.e. in the range $-\pi \le \omega < \pi$. Obtain a compact expression for $x[n]$.

d) **[1 *pt*]** Presuming ideal reconstruction conditions, indicate what sinusoidal frequencies (in Hertz) exist in $y_c(t)$, and indicate what their magnitudes are.

3. Consider the following Matlab code.

```
x=[4j  0  0  0  1  2j  3];
N=length(x);
X=fft(x); Y=zeros(1,N);
Y(1)=X(1); Y(N:-1:2)=X(2:N);
Z=conj(Y).*Y;
z=ifft(Z);
```

a) **[1 *pt*]** Without executing the code, find the result of `ifft(Y)`.

b) **[1,5 *pts*]** Without executing the code, find the result of `ifft(Z)`.

c) **[1 *pt*]** Without executing the code, explain why `z(1)= sum(x.*conj(x))` and explain why the $z[n]$ sequence is conjugate-symmetric.

4. Figure A represents the fundamental frequency (or pitch, in Hz) obtained from the singing of a female singer. The signal in Fig. A is available in vector `vbt` whose samples presume a sampling period of $\frac{512}{22050}\approx 23.22$ ms. The oscillation of the pitch around its average value (about 388.5 Hz) is called vibrato. In this exercise, we want to find the vibrato frequency using DFT analysis. Figures B and C represent the absolute value of the gain-normalized DFT using the following Matlab code in one case:

```
X=fft(vbt, N); % N=46, N2= N/2
plot([0:N2-1], abs(X(1:N2))/sum(vbt))
```

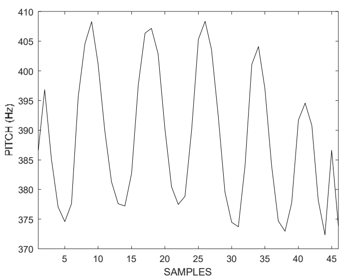
and using the alternative Matlab code in the other case:

(*continues*)

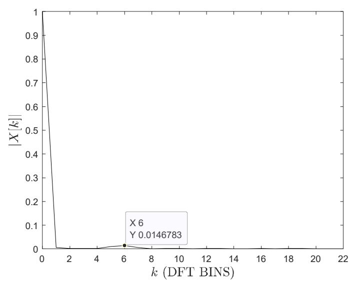
```
X = fft(vbt-mean(vbt)); % N=46, N2= N/2
plot([0:N2-1], abs(X(1:N2))/sum(vbt))
```

Both data tips in Figure B and C denote a local maximum for $k = 6$. Using an accurate frequency estimator, the local maximum on the DFT bin scale is found for $k \approx 5.6$.

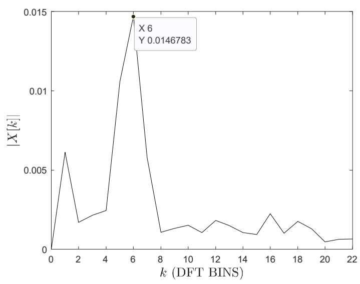
A



B



C



- **a)** **[1 *pt*]** Explain which code generates which figure, and why.
- **b)** **[1,5 *pts*]** Estimate the vibrato frequency in Hertz. Explain your reasoning.

  NOTE: the typical vibrato frequency is between 5 Hz and 8 Hz.

- **c)** **[1 *pt*]** We admit that the signal represented in Fig. A is real-valued. If prior to FFT transformation the signal is multiplied by a Hamming window, instead of the (default) Rectangular window, what is the expected impact of that on $|X[k]|$, and on the accuracy of the vibrato frequency estimation ?

5. Admit that you use an FIR adaptive filter of length 16 (i.e., the order is 15) in a system identification configuration. The 'black box' system consists of a second order all-pass system whose zero-pole diagram is represented in Figure A, and the adaptive filter starts from rest (i.e., all coefficients are zero at start). The sampling frequency is 8 kHz, the excitation is white Gaussian noise, adaptation is implemented using the LMS algorithm ($h_{n+1}[k] = h_n[k] + \mu\, e[n]x[n-k]$) and various adaptation step sizes (μ).

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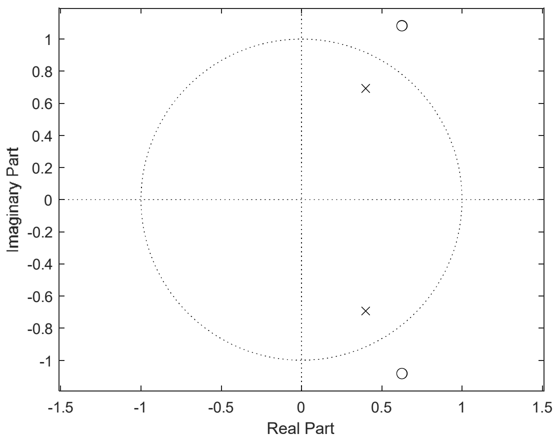
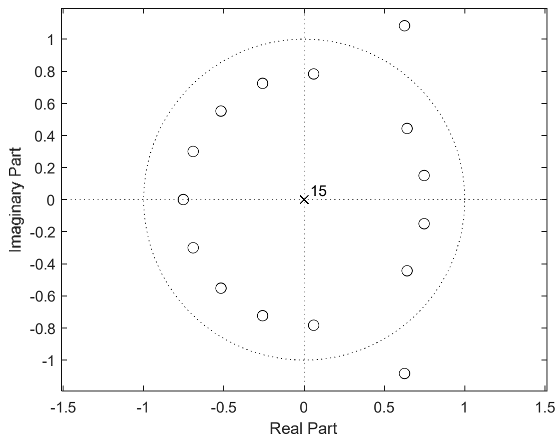
A

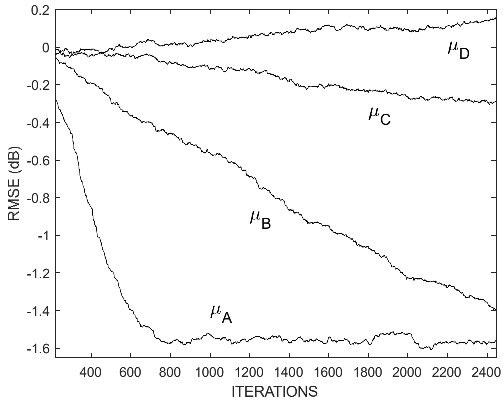


B



a) **[1,25 *pts*]** Sketch a block diagram of the adaptive system that is configured as system identification. Describe its main blocks.

b) **[1,25 *pts*]** The Root Mean Square Error (RMSE) of the adaption process is represented on the right-hand side for various values of μ: -0.025, 0.025, 0.05, 0.5. Based on the RMSE trends, what value do μ_A, μ_B, μ_C and μ_D represent ? Why?



c) **[1 *pt*]** After adaptation, the identified system has the zero-pole diagram represented in Figure B. How do you explain that the system in Figure A is identified as the system in Figure B ?

END