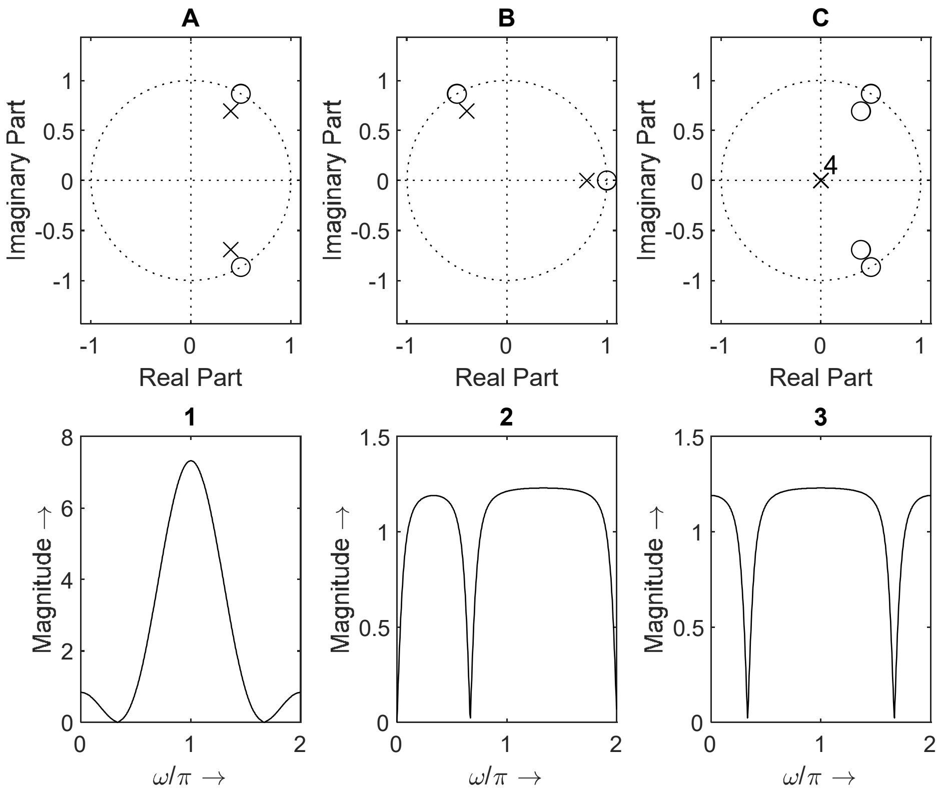
L.EEC/M.EEC
L.EEC025 - Fundamentals of Signal Processing

FIRST EXAM, JANUARY 19, 2024
Duration: 120 Minutes, closed book

NOTE: each question ***must*** be answered in a separate sheet; please write your name and Student order number on all sheets, please provide complete answers while trying to minimize paper usage.

1. Three causal discrete-time systems have the illustrated zero-pole diagrams A, B, and C, and the illustrated frequency response magnitudes 1, 2, and 3. Admit that the radius of all poles and zeros is either 0.8 or 1.0, and that the angles of all poles and zeros are multiples of $\pi/3$.



a) **[1,5 *pts*]** Match each zero-pole diagram (A, B, C) to the corresponding frequency response magnitude (1, 2, 3), and indicate the main supporting arguments.

b) **[1 *pt*]** Which of the represented systems (A, B, or C) are stable ? And which ones have a real-valued impulse response ? Why ?

c) **[1 *pt*]** Consider new systems formed by the following cascades: AB, AC, and BC. Which of these new systems have a linear phase response ? Why ?

d) **[1 *pt*]** «If $h_A[n]$ and $h_B[n]$ represent the impulse response of system A and system B, respectively, then $h_A[n]$ may be obtained by modifying $h_B[n]$ in a suitable way». Is this statement true or false ? If it is true, what is the suitable modification ?

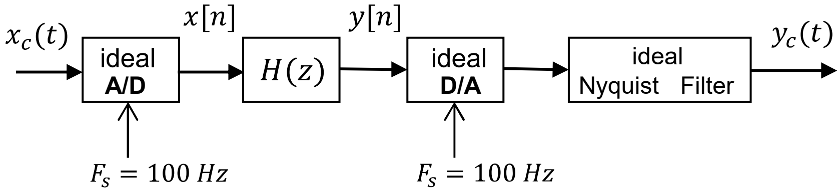
2. Consider system C as described in Prob. **1**.

a) **[1,5 *pts*]** Show that the transfer function of the system can be expressed as a cascade of two second-order sub-systems having real-valued coefficients. Sketch a canonic realization structure of the cascade of the two sub-systems.

(***continues***)

b) **[1 *pt*]** Admitting that the transfer function of the system is simply given by $H(z) = 1 - z^{-1} + z^{-2}$, obtain a compact expression characterizing the magnitude of the frequency response of the system, $|H(e^{j\omega})|$, and show that the gain is 1 when $\omega = 0$ rad., and is 3 when $\omega = \pi$ rad.

c) **[1 *pt*]** Consider the illustrated analog and causal discrete-time system whose transfer function is as suggested in **b)**. The sampling frequency is 100 Hz. The analog input signal is $x_c(t) = 1 + \sin(350\pi t)$. Notice that an *anti-aliasing* filter does not exist.



Find the sinusoidal frequencies of the discrete-time signal $x[n]$ in the Nyquist range, i.e. in the range $-\pi \le \omega < \pi$. Obtain a compact expression for $x[n]$.

d) **[1 *pt*]** Presuming ideal reconstruction conditions, indicate what sinusoidal frequencies (in Hertz) exist in $y_c(t)$, and indicate what their magnitudes are.

3. Consider the following Matlab code.

```
x=[1j  2  3j  4]; X=fft(x);
A=real(X); B=j*imag(X);
Y=A.*B;
y=ifft(Y)
Z=(X.*X-conj(X).*conj(X))/4;
z=ifft(Z)
```

a) **[1,5 *pts*]** Without executing the code, find and explain the result of `ifft(A)`, as well as the result of `ifft(B)`.

b) **[1,5 *pts*]** Without executing the code, find and explain the result of `ifft(Y)`.

c) **[1 *pt*]** Without executing the code, explain why `z=ifft(Z)=y=ifft(Y)`.

(***continues***)

4. Figure A represents the fundamental frequency (or pitch, in Hz) obtained from the singing of a female singer. The signal in Fig. A is available in vector `vbt` whose samples presume a sampling period of $\frac{512}{22050} \approx 23.22$ ms. The oscillation of the pitch around its average value (about 388.5 Hz) is called vibrato. In this exercise, we want to find the vibrato frequency using the autocorrelation function. Figures B and C represent the energy-normalized autocorrelation function using the following Matlab code in one case:

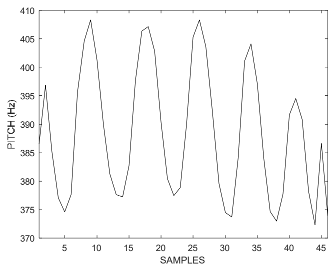
```
GAIN = sum(vbt.*vbt);
[xc lag] = xcorr(vbt, 20);
plot(lag,xc/GAIN)
```

and using the alternative Matlab code in the other case:

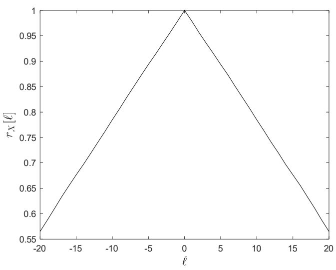
```
GAIN = sum((vbt-mean(vbt)).*(vbt-mean(vbt)));
[xc lag] = xcorr(vbt-mean(vbt), 20);
plot(lag,xc/GAIN)
```

The data tip in Figure C indicates $x = 8$, and $y \approx 0.68$.

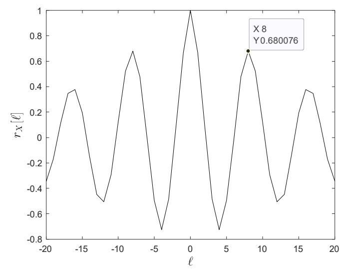
A



B



C



a) **[1 *pt*]** Explain which code generates which figure, and why.

b) **[1 *pt*]** Estimate the vibrato frequency in Hertz. Explain your reasoning.

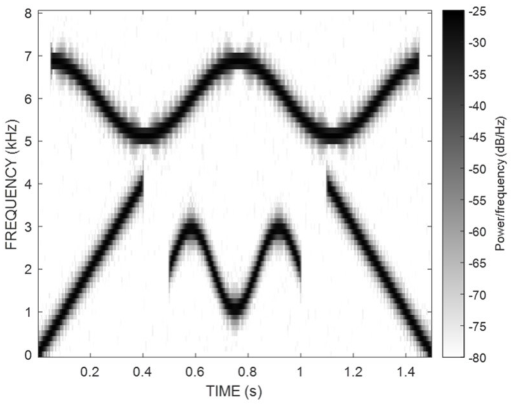
NOTE: the typical vibrato frequency is between 5 Hz and 8 Hz.

c) **[1,5 *pts*]** Admit that you want to obtain the same result of Fig. B, or C, using frequency-domain processing (i.e., using FFTs). Sketch a block diagram of that frequency-domain processing and describe each block, namely the size of each FFT.

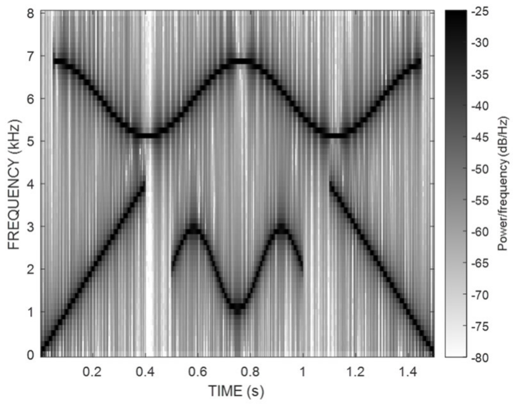
5. The spectral contents of an audio signal (FS=16 000 Hz) was analyzed using a sliding FFT (N=128), with 50% overlap between adjacent FFTs. Spectrograms A and B were obtained with two alternative windows: Rectangular and Hanning.

(***continues***)

A



B



a) **[1 *pt*]** What window has been used to generate spectrogram A ? What window has been used to generate spectrogram B ? Explain your reasoning.

Note: the blurred effects in the spectrograms reflect the impact of signal processing and not printer problems

b) **[1,5 *pts*]** Based on the observation of the spectrograms, describe the spectral contents of the audio signal.

c) **[1 *pt*]** Admit that the illustrated signal is filtered by two ideal filters: a low-pass filter, and a high-pass filter, both having 4.5 kHz cutoff frequency. Sketch the spectrogram of the signal at the output of each one of the two filters.

END