Material based on the paper: Neural Networks for Relational Data

- Deep networks (NNs with several layers) have some limitations:
 - lack of interpretability and explainability
 - somewhow limited to tabular representations (propositional)
 - graph networks can be limited to binary relations and also by data dimension
- Symbolic and structured representations are interpretable and allow for multiple levels of abstraction
- *Relational neural networks*: embed existing data relations to the network **structure** and learn parameters

Example:

Given: Set of instances \mathcal{F} , **Target** relation, relational data set $(\mathbf{x}, y) \in \mathcal{D}$; **Construct** (structure learning): \mathbf{R}_j , relational random walk rules (relational feature describing the network structure of \mathcal{N}); **Train** (parameter learning): w_j , rule weights via gradient descent with rulebased parameter tying to identify a sparse set of network weights of \mathcal{N}

Example. The movie domain contains the entity types (variables) Person(P), Movie(M) and Genre(G). In addition there are relations (features): Directed(P, M), ActedIn(P,G) and InGenre(M,G). The domain also has relations for entity resolution: $SamePerson(P_1, P_2)$ and $SameGenre(G_1, G_2)$. The task is to predict if P_1 worked under P_2 , with the target predicate (label): $WorkedUnder(P_1, P_2)$. Using random walks:

Example (continued). The body of the rule

 $\texttt{ActedIn}(\texttt{P}_1,\texttt{G}_1) \land \texttt{SameGenre}(\texttt{G}_1,\texttt{G}_2) \land \texttt{ActedIn}^{-1}(\texttt{G}_2,\texttt{P}_2) \land$

 $\texttt{SamePerson}(\texttt{P}_2,\texttt{P}_3) \land \texttt{ActedIn}^{-1}(\texttt{P}_3,\texttt{M}) \land \texttt{Directed}(\texttt{M},\texttt{P}_4) \Rightarrow \texttt{WorkedUnder}(\texttt{P}_1,\texttt{P}_4)$

can be represented graphically as

 $\mathsf{P}_1 \xrightarrow{\texttt{ActedIn}} \mathsf{G}_1 \xrightarrow{\texttt{SameGenre}} \mathsf{G}_2 \xrightarrow{\texttt{ActedIn}^{-1}} \mathsf{P}_2 \xrightarrow{\texttt{SamePerson}} \mathsf{P}_3 \xrightarrow{\texttt{ActedIn}^{-1}} \mathsf{M} \xrightarrow{\texttt{Directed}} \mathsf{P}_4.$

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Relational Neural Networks

Network structure is built from output to input:



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- Step 1: use path-constrained random walk to build $R_j, J = 1, 2, ..., M$ random walks \rightarrow relational features
- paths are *lifted*: logical variable generalizes groundings

- Step 2: Output layer:
 - ▶ for the **Target**, which is also the head $h(A_h \text{ in the net})$ in all R_j , introduce an output neuron called the target neuron \rightarrow one-hot encoding can be used to code the target labels
 - The target neuron uses the softmax activation function (for multinomial probability distribution on the multiple classes)

Step 3: Combination Rules layer: target is connected to M lifted rule neurons A_j, j = 1,..., M → lifted relational random walks (R_j, w_j)
each R_j is a conjunction of predicates:

$$Q_1^j(X,.) \wedge \dots \wedge Q_L^j(.,Z) o {\it Target}(X,Z), j=1,\dots,M$$

each lifted A_i combines the different groundings

train the network to learn parameters

- Step 4: Grounding layer: for each instance of A_j introduce a ground rule neuron A_{ji}
- A_{ii} is only activated if all ground term of its conjunction is true
- these neurons are connected to the input layer which consists of the facts that compose each R_j

Example:



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- special case of a *convolutional* neural network (CNN)
 - fact-grounding layer edges are equivalent of convolution (reduce dimensions)
 - combination rules layer: pooling
 - fully-connected layer (output): softmax

Limitations:

- same weights to distinct ground predicates
- binary predicates
- no connection between neurons in the same layer
- random walks may not produce representative paths

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- why RNN if paths can be searched using ILP?
- why the RNN is built in layers?